WRIGHT STATE UNIVERSITY | DEPARTMENT OF MATHEMATICS AND STATISTICS

COLLOQUIUM

Speaker:	**Dr. Reza Akhtar, Miami University**		
Title:	**Quasigroups, generalized associativity, and automated theorem-proving**		
Date:	**Friday, December 1, 2017**		
Room/Time:	**Meet-n-Greet:**	**2:30 p.m.**	**Room 222 MM**
	Talk:	**3:00 p.m.**	**Room 224 MM**

ABSTRACT:

An important property of groups, introduced early in the study of Abstract Algebra, is the so-called *Latin Square Property*, namely that the Cayley table of a finite group is a Latin square – or more generally, that for any group G and $a \in G$, the left- and right-multiplication maps L_a and R_a on G, defined (respectively) by $L_a(x) = ax$ and $R_a(x) = xa$, are bijections. If one replaces the usual group axioms by only the Latin square property, one obtains a weaker structure known as a *quasigroup*. Quasigroups are in general more plentiful and more difficult to study than groups; however, it has proven useful to study the varieties of quasigroups defined by various algebraic conditions. For instance, every group is an associative quasigroup, and it is not difficult to show that the converse also holds; thus, the variety of quasigroups satisfying associativity coincides with the variety of groups.

The associative law is, of course, the identity $x(zy) = (xz)y$, which can be expressed succinctly in terms of multiplication maps as $L_x R_y\,(z) = R_y\,L_x(z)$, or simply $L_x R_y = R_y L_x$. Noting that the two sides of the identity are mirror images of each other, a natural question then arises: what other sorts of varieties of quasigroups are defined by similarly "symmetric" identities involving left and right multiplication maps? For instance, if one assumes the identity $L_x R_y L_z L_w = L_w L_z R_y L_x$ in a quasigroup, does associativity hold? Does commutativity hold? Surprisingly, it turns out that even when one considers symmetric identities of arbitrary length in left and right multiplication maps, there are exactly eight varieties of quasigroups so defined, and one can describe completely the various inclusion relations among them. Although all the proofs were eventually written down by hand, the automated theorem-prover `Prover9` and its companion model-builder `Mace4` were indispensable in building an understanding of the relationships among the various identities. In this talk, I will discuss this result (including a sampling of proofs) and describe the role played by automated theorem-proving in the analysis.

SPEAKER BIO:

Reza Akhtar received his Ph.D. in May 2000 from Brown University. In August 2000, he joined the faculty of Miami University, where he is now Professor of Mathematics. His dissertation research was in the area of algebraic cycles on varieties, and he has published several papers in this area. While working as a seminar director at the SUMSRI program, he became interested in various other problems at the intersection of algebra and combinatorics, among them various questions related to quasigroups and automatic theorem-proving.