DEPARTMENT OF MATHEMATICS AND STATISTICS

COLLOQUIUM

Speaker: **Dr. Joseph Rosenblatt, Indiana University-Purdue University, Indianapolis**

Title: **Convergence and Divergence of Averages**

Date: **Friday, April 14, 2017**

Room/Time:	**Meet–n–Greet:**	**2:30 p.m.**	**Room 222 MM**
	Talk:	**3:00 p.m.**	**Room 224 MM**

Host: **Dr. Ayşe Şahin**

ABSTRACT:

In many applied and theoretical contexts, we have a sequence of data that is a sample from functions $(f_k(x) : k \geq 1)$ where $x \in X$ for some probability space X. There can be even more structure in that there is a map τ of X and a fixed function f such $f_k(x) = f(\tau^k x)$ for all $k \geq 1$. In probability or statistics, the functions f_k can often be assumed to be independent and identically distributed (IID). But in statistical mechanics, and in ergodic theory and dynamical systems, the functions f_k are usually correlated and so not independent.

Given the data sequence (f_k), there is a classical Ergodic Theorem, extending the Law of Large Numbers (LLN): the averages $A_n f = \frac{1}{n}\sum_{k=1}^{n} f_k$ converge in norm and also pointwise almost everywhere to a limit L, which is a constant in the ergodic (and IID) case.

We want to know if the LLN or more generally the Ergodic Theorem extends to where we are taking averages of only part of the data sequence. For example, what if we only sample the data sequence so infrequently that $A_n = \frac{1}{n}\sum_{k=1}^{n} f\left(\tau^{2^k} x\right)$? In the IID case, we would have the LLN again. But in the correlated case these averages diverge a.e. for most functions.

Short of sampling the data sequence rarely, we could be at least sample total blocks of terms, but perhaps infrequently. Then the averages would be *moving averages* and have the form $M(v_n, L_n)f = \frac{1}{L_n}\sum_{k=v_n+1}^{v_n+L_n} f \circ \tau^k$, where $v_n \in \mathbb{Z}$ and $L_n \in \mathbb{Z}^+$. When do these converge a.e. for all functions? Even in the IID case, this question does not have a simple answer!

We will first consider criterion for a.e. convergence of moving averages from a global perspective where we look for convergence to happen all of the time: for all maps and all functions. We will then look at moving averages from a local perspective, where we fix the map and consider various functions, and vice versa fix the function and vary the map. We finish with some remarks about Cesàro averages along rare subsequences. Our methods use techniques from harmonic analysis, probability theory, ergodic theory, and descriptive set theory.

ABOUT THE SPEAKER:

Joseph Rosenblatt is a Professor of Mathematics at Indiana University-Purdue University Indianapolis. Before coming to IUPUI in 2014, he was a Professor of Mathematics at the University of Illinois (1994 to 2014), a faculty member at both Ohio State University and the University of Missouri-Columbia (1974-1994), a postdoctoral faculty member at the University of British Columbia (1972-1974), and a graduate student at the University of Washington (1968-1972). Also, Professor Rosenblatt was a Program Officer at NSF (2006-2008).

Professor Rosenblatt's research interests include harmonic analysis, both classical and abstract, dynamical systems, probability theory, functional analysis, and applications of analysis in science and engineering. He has written over 100 research articles and has held numerous grants to support his research. He has been the advisor of a dozen Ph.D. students at Ohio State University and the University of Illinois.