Optimal taxation and heterogeneous oligopoly

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Abstract. The paper examines the effects of firm-specific taxes on an oligopoly composed of firms of unequal sizes, holding different conjectures, operating with non-identical costs. Four types of effects are analysed: on profits, on industry concentration, on total tax revenue, and on social welfare. We derive (i) the revenue-maximizing taxes, (ii) the welfare-maximizing taxes, and (iii) welfare-maximizing taxes subject to a revenue constraint.

Fiscalité optimale et oligopole hétérogène. Ce mémoire examine les effets de taxes spécifiques à une firme sur un oligopole composé de firmes de tailles inégales, ayant des conjectures différentes, et opérant avec une structure de coûts qui n’est pas identique d’une firme à l’autre. Quatre types d’effets sont analysés: effets sur les profits, sur la concentration industrielle, sur les revenus fiscaux globaux, et sur le niveau de bien-être social. L’auteur dérive (i) les impôts qui maximisent le revenu, (ii) les impôts qui maximisent le bien-être, et (iii) les impôts qui maximisent le bien-être sous une contrainte de revenu.

1. INTRODUCTION

The rapid growth of the theoretical literature on the effects of taxes on oligopoly in recent years has been stimulated and shaped by several confluent factors. First, although imperfect competition is clearly the preponderant industry structure, possibly with important macroeconomic consequences, our knowledge of it is quite sketchy compared with what we know about the less realistic structures of perfect competition and pure monopoly. Second, the familiar tools of price theory (in contrast to, say, the case study approach) have been found to remain quite potent in the study of oligopoly, making this area more inviting to a larger number of researchers. Last, and most tantalizing, already obtained results (e.g., taxes could raise

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profits) hint at many more unexpected, even counterintuitive phenomena awaiting discovery.

This paper contributes to this emerging paradigm. It moves beyond the existing literature in two directions: one concerns the restrictions imposed on the model; the other concerns the questions addressed.

First, extant works on the effects of taxes on price and profits have focused almost exclusively on the case of symmetric oligopoly (Katz and Rosen 1985; Stern 1987; Myles 1987). By contrast, we examine an oligopoly in which firms differ not only in (possibly non-linear) costs and in their conjectures, but also in the tax/subsidy rates to which they are subject. The last extension is especially significant. Although few taxes are expressly firm specific, there are important government interventions that are quite closely related to a firm’s cost, conduct, and size. For instance, firms participating in the federal procurement market in the United States must comply with various affirmative action programs (see, e.g., Dung and Premus (1990)). As these firms are usually large, the requirement is prima facie size discriminatory. Likewise, in nearly every country populist sentiments (aided in no small measure by effective lobbying by small business groups) are often translated into a broad array of direct and indirect programs with the express intention of helping small businesses. In the United States, for example, firms that employ few workers are exempted from the minimum wage law and other requirements, such as those dictated by the Americans with Disabilities Act.

Second, going beyond the occasional attention given to the welfare effect of taxes on oligopoly (Brander and Spencer 1985; Katz and Rosen 1985; Myles 1987), we seek to determine both the revenue-maximizing tax and the welfare-maximizing tax for this kind of industry. In this regard, our analysis extends the work of Myles (1987), who assumed identical oligopolists, all of whom adopt Cournot conjectures and have constant marginal cost.

Third, we pay special attention to the connection between commodity taxation and industry concentration. While some models have touched on this question, their results are not particularly useful, since they almost invariably assume that the oligopoly is homogeneous, which implies that concentration is but the inverse of the number of firms in the industry. As the interest in the degree of industry concentration stems mainly from its being a presumptive indicator of market power, the dubious inference could then be drawn that parametric changes (such as taxes)

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1 Notable exceptions that allow for non-identical firms are Dixit and Stern (1982), Levin (1985), Dixit (1986), Diericks et al. (1988), and particularly Seade (1985).

2 And of course taxes are de facto discriminatory, since the variation of tax rates across state/local, coupled with the fact that market shares also vary across regions, resulting in firm-specific taxes.

3 It must be noted, however, that in the voluminous literature on optimal commodity taxation (see, e.g., Atkinson and Stiglitz 1980) the optimality is defined in the context of many goods (including labour) and many heterogeneous consumers. By contrast, the optimality we are seeking is with regard to the tax structure to be imposed on a heterogeneous oligopoly producing a homogeneous good selling to a representative consumer.

4 Myles’s model has a perfectly competitive sector besides the oligopoly, however; hence his entire model is very much like a duopoly with one ‘firm’ consisting of his identical oligopolists and one ‘firm’ acting competitively.
can affect market power only in so far as they induce entry or exit. By tracing out the effect of a change in tax rates on the concentration of a heterogeneous oligopoly with a fixed number of firms and contrasting it with the effect on welfare, we hope to delineate the role of heterogeneity in the link between industry concentration and welfare, especially in a changing economic environment.

Our analysis yields a number of interesting results. First, we discover that a tax increase could perversely cause a firm to expand its output in an asymmetric duopoly – something impossible in a symmetric oligopoly. Second, contrary to a common assumption, neither revenue nor welfare maximization necessarily calls for a heavier tax on the larger firms. Third, the effect of either revenue- or welfare-maximizing taxes on industry concentration is ambiguous, even in its sign.

The paper is organized as follows. In section II the basic model is laid out. Section III examines the effects of taxes on industry concentration, tax revenue, and social welfare. A summary and conclusions are offered in section IV.

II. THE BASIC MODEL

Consider a duopoly with two non-identical firms, 1 and 2.\(^5\) Let \(y_f\) and \(C_f'(y_f, t_f)\) denote the output and the tax-inclusive total cost function of the firm \(f (f = 1, 2)\), respectively, with \(t_f\) being the firm-specific tax. It is assumed that \(C_{yy}^f > 0\), \(C_{yt}^f > 0\), \(C_{yt}^f \geq 0\), \(C_{yt}^f > 0\), and \(C_{t}^f \geq 0.\)\(^6\) With no loss of generality, suppose \(C_y^1 \leq C_y^2\) for all level of outputs; that is, firm 1 is more efficient than firm 2.\(^7\)

Let \(P(\Sigma_i y_i)\) be the inverse market demand, with \(P' < 0\). The profit of firm \(f\) is

\[
\Pi_f = P y_f - C_f'(y_f, t_f). \tag{1}
\]

Oligopolistic behaviour is captured by the firm’s conjectural variation parameter \(k_f = d(\Sigma_i y_i)/dy_f\) which is taken to represent the belief held by firm \(f\) with respect to the industry’s reaction to a change in its own output. We allow \(k_1 \neq k_2\) but assume that they are otherwise invariant (to output and tax, in particular). As is well known, for a duopoly, \(k_f\) is bounded between 0 and 2, with the lower value corresponding to perfectly competitive behaviour and the higher value indicating total collusion. The Cournot value is 1.\(^8\) The objections to the conjectural variations approach are not hard to find, but so are its advantages (see, e.g., Kamien and Schwartz 1983). Since most of the results obtained here turn out to be sensitive to \(k_f\), it is worthwhile (especially in drawing policy implications) to keep in mind

\(^5\) The analysis can be easily extended to the case of \(n > 2\) firms. Details are available from the author.

\(^6\) \(C_y^f = dC^f /dy_f\), \(C_t^f = dC^f /dt_f, C_yy^f = d^2C^f /dy_f^2, C_{yt}^f = d^2C^f /dy_f dt_f,\) and \(C_{t}^f = d^2C^f /dt_f^2.\)

\(^7\) This interpretation is admittedly somewhat narrow, since firm 1 could incur a larger fixed cost.

\(^8\) There is a controversy as to whether or not the permissible values of conjectural variations can be further narrowed down by some consistency rule. Such is the idea of consistent conjectural variations (CCV) broached by Bresnahan (1981) (see also Perry 1982). Other authors (Laitner 1980; Kamien and Schwartz 1983; and especially Boyer and Moreaux 1983a, b) have found, however, that under general conditions of costs, demand, and conjectural variations functions, the CCV requirement does not necessarily limit the permissible range of conjectural variations.
how conjectural variations might be interpreted. The current consensus (see, e.g., Cubbin 1983; Kalai and Stanford 1985; Riordan 1985) seems to be that \( k_f \) can be related to firm \( f \)'s expectation of retaliation by its competitors against its actions.\(^9\)

The first-order condition for profit maximization is

\[
P + k_f y_f P' - C_y^f = 0,
\]

and the second-order condition is

\[
k^2_f y_f P'' + 2k_f P' - C_y^{ff} < 0,
\]

which can be rewritten as

\[
k_f (P' + k_f y_f P'') + (k_f P' - C_y^{ff}) < 0.
\]

To satisfy the second-order condition and ensure stability, we assume that \( k_f (P' + k_f y_f P'') < 0;^{10} \) that is, the perceived marginal revenue curve is steeper than the market demand curve, or perceived demand is weakly convex. To ease notation, we denote

\[
\alpha_f \equiv (P' + y_f k_f P'')/(k_f P' - C_y^{ff}) > 0,
\]

\[
\beta_f \equiv C_y^{ff}/(k_f P' - C_y^{ff}) < 0,
\]

and

\[
\gamma_f \equiv \alpha_f \beta_f - \alpha_f \beta_f = -\gamma_f.
\]

In terms of more familiar elasticities, it can be easily shown that

\[
\alpha_f = \frac{s_f + s_f^2 k_f E}{s_f k_f - \mu_f (s_f k_f + \epsilon)}
\]

\[
\beta_f = \frac{\eta_f (s_f k_f + \epsilon) \Sigma y_i / t_f}{s_f k_f - \mu_f (s_f k_f + \epsilon)},
\]

where \( s_f \equiv y_f / \Sigma y_i \) is the market share of firm \( f \), \( \epsilon \equiv (\Sigma y_i) P' / P \) is the elasticity of demand, \( E \equiv (\Sigma y_i) P'' / P' \) is the elasticity of the slope of market demand,\(^{11} \) \( \mu_f \equiv y_f C_y^{yy} / C_y^{ff} \) is the elasticity of firm \( f \)'s marginal cost with respect to its output,\(^{12} \) and \( \eta_f \equiv t_f C_y^{yy} / C_y^{ff} \) is the elasticity of firm \( f \)'s marginal cost with respect to its tax.

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9 See also Klemperer (1987) for an interesting explanation in terms of consumer switching costs.
10 See Seade (1980). Actually, we must also have \( (k_f P' - C_y^{ff}) < 0 \). However, this condition is automatically fulfilled by our already-made assumptions that \( P' < 0 \) and \( C_y^{ff} > 0 \).
11 The value of \( E \) (positive for concave demand, negative for convex demand) plays a critical role not only in Seade’s (1981, 1985) analysis but also in Joan Robinson’s (1935) pioneering work on imperfect competition. Robinson called it the ‘adjusted concavity’ of demand.
12 Note that \( \mu_f \) is the inverse of the elasticity of firm \( f \)'s competitive supply curve.
Lemma 1. A rise in taxes never increases industry output.

Proof. Totally differentiating the first-order condition, assuming that the conjectural variations are constant, yields

\[ \alpha_f \sum_i dy_i + dy_f - \beta_f dt_f = 0. \] (5)

Summing (5) over two firms, we obtain

\[ (\sum_i \alpha_f)(\sum_i dy_i) + \sum_i dy_i - \sum_i \beta_i dt_i = 0. \] (6)

Hence,

\[ \sum_i dy_i = \sum_i \beta_i dt_i / \Delta, \] (7)

where \( \Delta \equiv 1 + \sum \alpha_i > 0 \). Since \( \beta_i < 0 \), we must have \( \sum_i dy_i < 0 \).

Note that \( \alpha_f \) bears a simple relation to the slope of firm \( f \)'s reaction curve in the output space. If that slope is written \( R_f \in (-1, 0) \), then \( \alpha_f \equiv -R_f / (1 + R_f) \).\(^{13}\)

Lemma 1 affirms that at the industry level the consequence of interfirm asymmetry cannot possibly be so perverse as to reverse the familiar sign of the effect of taxes on output. This impossibility also holds for an individual firm's output if the oligopoly is symmetric, but not necessarily if the oligopoly is asymmetric. In particular,

Proposition 1. Tax changes will decrease the output of firm \( f \) if and only if

\[ dt_f / dt_j > \alpha_f \beta_j / (1 + \alpha_j) \beta_f \geq 0. \] (8)

Proof. Substituting (7) back into (5) yields

\[ dy_f = \beta_f dt_f - \alpha_f (\sum_i \beta_i dt_i) / \Delta, \] (9)

from which (8) follows.

Under what circumstances would an increase in the tax on firm \( f(dt_f > 0) \) cause its output to expand? Proposition 1 asserts that this perversity would occur if \( 0 < dt_f / dt_j < \alpha_f \beta_j / (1 + \alpha_j) \beta_f \). In words, an increase in the tax on firm \( f \) would raise its output if there is also a substantial increase in the tax on the other firm. The intuition is rather simple: subject to a sufficiently large increase in its tax, the other firm could reduce its output so much that the ensuing increase in the

\(^{13}\) The signs of these elasticities can be traced to the second-order and stability conditions.
residual demand facing firm $f$ would offset firm $f$’s own higher tax, causing firm $f$ to expand its output.

Similarly, an increase in the subsidy to firm $f$ would (perversely) decrease its output if and only if there is a substantial increase in the subsidy to firm $j$ as well.

Let us now explore the effect of taxes on profits. Totally differentiating the profit expression (10) gives

$$d\Pi^f = (P - C^f_v)dy_f + y_f dP - C^f_t dt_f. \quad (10)$$

Firm $f$ benefits proportionately to its mark-up from increases in its own production, is affected proportionately to its output by price changes, and experiences a direct effect of the change in its own tax rate on its total costs. Making use of the first-order condition and noting that $dP = P'(\Sigma_i dy_i) = P'\Sigma_i \beta_i dt_i/\Delta$, we can rewrite (10) as

$$d\Pi^f = \left\{ -C^f_t - \frac{y_f P'\beta_f [k_f(1 + \alpha_f) - 1]}{\Delta} \right\} dt_f + \left\{ \frac{y_f P'\beta_f [k_f \alpha_f + 1]}{\Delta} \right\} dt_f. \quad (11)$$

Equation (11) suggests the profit effect of changes in taxes can be broken down into three components. First, the change in $t_f$ will affect firm $f$’s own profit (a) directly by increasing the tax collected at the initial output level, and (b) indirectly through its effect on the new equilibrium, which will entail a new output. The total effect of $t_f$ on firm $f$’s own profit could be of either sign. And second, firm $f$’s profit will also be influenced by the equilibrium-altering tax imposed on firm $j$. This effect will always be positive. Taken together, the overall effect set off by a change in the tax configurations on one firm’s profit could be positive or negative.\(^\text{14}\)

The intuition is that initially the firms could be over- or under-taxed relative to the level that would maximize industry profit (given that the firms are not totally collusive). If they were overtaxed, a tax cut will make the industry more profitable as the reduction in tax payments will exceed any fall in gross profits due to de facto more aggressive competition. Conversely, if the firms were undertaxed, a tax increase will boost the firms’ profits by facilitating a more nearly joint profit maximization outcome, the benefits of which will more than offset the larger tax bills.

To isolate the effect of asymmetry, rewrite (11) as

$$d\Pi^f = -[C^f_t + (y_f P'\beta_f k_f/\Delta)]dt_f + (y_f P'/\Delta)(\beta_f dt_f + \beta_j dt_j)$$

$$- (y_f P'/\Delta)(\alpha_f \beta_f dt_f - \alpha_f \beta_j dt_j). \quad (11')$$

The first two terms on the RHS of (11’) are definitely negative and positive, respectively, whether or not there are any interfirm dissimilarities. The third term, which vanishes if firms are identical, is positive or negative (and thus strengthens or

\(^{14}\) This provides the rationale for ‘cost raising strategies,’ as suggested by Salop and Scheffman (1987).
weakens a firm’s chance of benefiting from a tax increase) depending on how
dissimilar the firms are from each other. Absent policy discrimination (i.e., assuming
\( dt_f = dt_j \)), this term acquires the sign of \( \gamma_f(\equiv \alpha_f \beta_f - \alpha_f \beta_j) \). Simple substitution
shows that \( \gamma_f > 0 \) if and only if \( C_f^d/[1 + (s_f k_f/E)] < C_j^d/[1 + (s_j k_f/E)] \). In the case
of linear demand, constant (but not necessarily identical) marginal costs, and
specific tax, this condition reduces to simply \( s_f k_f < s_j k_f \). Thus, the larger and/or more
collusive firm is less likely to benefit from asymmetry than the smaller and/or less
collusive firm, ceteris paribus. This result dovetails nicely with that obtained by
Stigler (1950) and Salant et al. (1983), whereby a horizontal merger might benefit
firms that elect not to participate in the merger.

III. EFFECTS OF TAXES ON CONCENTRATION, TAX REVENUE,
AND WELFARE

1. Tax and industry concentration
Notwithstanding its pivotal role in anti-trust decisions, the degree of industry con-
centration has rarely been the subject of close scrutiny in the theoretical literature,
especially in the way it could be affected by commodity taxation.\(^{15}\) This omis-
sion is perhaps justified if the oligopoly is homogeneous, in which case taxes can
change concentration only via entry or exit, about which plenty has been said in
the literature. Put differently, taxes are concentration neutral if the oligopoly is
homogeneous and the population of firms is constant. Now, if the oligopoly is
heterogeneous, taxes can still affect concentration through induced entry and exit.
But more importantly, if the oligopoly is heterogeneous, taxes can also influence
concentration without changing the number of firms, by altering the relative market
shares.

**Proposition 2.** If initially market shares are unequal, then tax changes will be
concentration neutral if and only if

\[
\frac{dt_f}{dt_j} = \frac{\beta_f(s_f + \alpha_f)}{\beta_f(s_j + \alpha_f)}. \tag{12}
\]

**Proof.** Let concentration be measured by the Hirschman-Herfindahl index \( H \); then
\( dH = 2 \Sigma_i s_i ds_i \), where \( ds_f = d\Sigma_i y_i - s_f d(\Sigma_i y_i)/\Sigma_i y_i \). Straightforward substitution,
making use of \( d\Sigma_f \) found in (9), yields

\[
dH = 2 \Sigma_i (s_i - s_j)(s_j + \alpha_j) \beta_i dt_i / \Delta \Sigma_i y_i,
\]

from which proposition 2 immediately follows.

\(^{15}\) Several useful contributions have recently been made. See, for example, Farrell and Shapiro
(1990a, b), Daughety (1990). None of them, though, has examined the link between concentration
and commodity taxation as we do here.
One corollary of proposition 2 is that, assuming $s_f > s_j$ initially, a uniform, non-discriminatory change in the tax rates will increase (decrease) concentration if and only if the inverse $\beta$-weighted sum of market share and $\alpha$ of the larger firm is greater (smaller) than that of the smaller firm; that is, $(s_f + \alpha_f)/\beta_f > (<)(s_j + \alpha_j)/\beta_j$.

If, for example, the two firms are identical (in the sense that $\alpha_f = \alpha_j$ and $\beta_f = \beta_j$) except for their market shares, then a uniform change in tax rates would make the large firm larger and the small firm smaller. Also, it may be worth pointing out that the $\text{HHI}$ will definitely be affected if only one firm’s tax is altered.

In any event, our analysis shows that a change in $\text{HHI}$ could be triggered by a change in the tax configuration, although there may be no change in the behaviour of the firms (as reflected by their conjectural variations). To be sure, current policies (such as the merger guidelines adopted by the U.S. Department of Justice) are not cognizant of possible changes in $\text{HHI}$ (owing to, say, interfirm growth rate differential) that are not caused by changing firms’ attitudes toward rivals. Still, our results serve as a useful reminder that changes in taxes may affect the $\text{HHI}$ as well.

2. Taxes and tax revenue

Theorists prefer to think of commodity taxes as but a means to correct market distortions or to finance some well justified public activities. In reality, however, hardly any tax is chosen, and its rate set, without the sheer size of the anticipated revenues’ being a significant if not determinative factor in the deliberation. In this section we examine a tax imposed on an oligopoly for the sole purpose of raising government revenue, the larger the better.

For simplicity let us assume henceforth that the tax is per unit of output, such as an excise tax; hence,

$$G = G(t_1, t_2) = \Sigma_i t_i y_i. \quad (13)$$

Making use of (12), we obtain

$$dG = \left\{ y_1 + \frac{(1 + \alpha_2) \beta_1 t_1}{\Delta} - \frac{\alpha_2 \beta_1 t_2}{\Delta} \right\} dt_1 + \left\{ y_2 + \frac{(1 + \alpha_1) \beta_2 t_2}{\Delta} - \frac{\alpha_1 \beta_2 t_1}{\Delta} \right\} dt_2, \quad (14)$$

or, more compactly,

$$dG = \Sigma_i \left\{ y_i + [\beta_i t_i + \alpha_j \beta_j (t_i - t_j)]/\Delta \right\} dt_i. \quad (14')$$

The term inside the brackets in (14') is composed of three elements representing the three channels through which an increase in firm $f$’s tax rate will affect the total tax revenue. First, such a tax hike will immediately boost the revenue collected at the initial output. Second, as the firm will subsequently reduce its output in response to the higher tax, the total tax revenue will, as a result, decline. Finally, and characteristic of oligopoly, there will be a feedback from the other firm’s output adjustment. This oligopolistic repercussion, captured by the last term inside
the brackets, will nudge the total tax revenue upward again. An examination of
(14') reveals that there will be no such repercussion of initially \( t_1 = t_2 \).

The revenue-maximizing tax pair can be solved for by setting the coefficients
inside the curly brackets in (14) simultaneously equal to zero:

\[
-(1 + \alpha_2)\beta_1/\Delta) t_1 + (\alpha_2\beta_2/\Delta) t_2 = y_1 \tag{15a}
\]

\[
(\alpha_1\beta_1/\Delta) t_1 - [(1 + \alpha_1)\beta_2/\Delta] t_2 = y_2. \tag{15b}
\]

Solving (15a-b) by Cramer’s rule, making use of the first-order conditions (2),
and letting \( c_f^j \equiv C_f^j - t_f \) denote the pre-tax marginal cost of firm \( f \), we obtain the
firm-specific revenue-maximizing tax rates:

\[
t_f^M = \left( \frac{(P - c_f^j)/k_f P'}{(1 + \alpha_f)\beta_j/\Delta + (1/k_f P')} \right) /\Psi
\]

\[+ \left( \frac{(P - c_f^j)/k_j P'}{(1 + \alpha_j)\beta_j/\Delta} \right) /\Psi. \tag{16}
\]

where

\[
\Psi \equiv \left( (1 + \alpha_1)\beta_2/\Delta + (1/k_2 P') \right) (1 + \alpha_2)\beta_1/\Delta + (1/k_1 P') - (\alpha_1\alpha_2\beta_1\beta_2/\Delta^2) > 0.
\]

As expected, if the firms are identical they should be taxed at the same rate.
More generally, \( \partial t_f^M / \partial c_f^j < 0 \) and \( \partial t_f^M / \partial c_f^j < 0 \), indicating that the tax on one
firm should be higher not only when its marginal costs fall, but also when the
marginal costs of its rival fall. Put differently, the revenue-maximizing tax rates
vary inversely with the industry’s average marginal costs. The intuition behind this
result is straightforward: the lower the industry’s average marginal cost, the heavier
the tax burden the industry could bear, allowing higher rates for the revenue-raising
tax system.

The effect of the revenue-maximizing tax on industry concentration is am-
biguous. In particular, contrary to a common presumption (see, e.g., Clarke 1988),
the revenue-maximizing tax structure is not necessarily progressive. The following
numerical example will suffice to demonstrate this possibility.

Suppose the market demand function is \( P = 10 - (y_1 + y_2) \) and choose \( C_1^1 = 1 \) and
\( C_2^2 = 2 \). Further assume that firm 2 adopts Cournot conjecture \( k_2 = 1 \). For different
values of \( k_1 \), table 1 reports the revenue-maximizing tax rates.\(^16\) This example demonstrates that if the low-cost firm (firm 1) adopted a relatively competitive
behaviour, it should be assessed a lower tax rate than the high-cost firm, given that
the authority’s sole intention is to maximize the total tax revenue. Conversely, if
the low-cost firm was strongly collusive, it should be more heavily taxed. Indeed,
in the latter case the tax could be so discriminating against the low-cost firm that
this firm could end up with a smaller market share. The intuitive for this result is
quite simple. Since tax revenue depends primarily on the industry output, its rate
should be lower for the firm that produces more, regardless of whether this high

\(^{16}\) Given the parameters of the example, firm 1 is not viable if \( k_1 < 0.4 \).
TABLE 1
Revenue maximizing taxes*

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<th>$k_1$</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$y_1^*$</th>
<th>$y_2^*$</th>
<th>$\delta^*$</th>
<th>HHI</th>
<th>HHI*</th>
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*Asterisks denote after-tax values.

output is due to the firm’s low costs or to its weak collusive attitude. Hence, firm 1, which is more efficient, should be favoured if it is also less collusive. But if this firm is acting in a way that is strongly collusive (and therefore tends to hold back its output) then the other firm, though less efficient, should receive a more preferential tax treatment.

For the numerical example given, we also compute the Hirschman-Herfindahl index before and after the revenue-maximizing tax. In either case, this index is U-shaped when plotted against $k_1$. More interestingly, an examination of the change due to this tax ($\Delta$(HHI) ≡ HHI* – HHI) reveals a range of conjectural variations in which this tax makes the industry less concentrated than before. For an apparently wider range of conjectural variations, though, this tax increases concentration.

3. Taxes and welfare
To avoid complications due to income and distributional effects (among consumers) we assume the consumption of this good is by a representative consumer whose utility function is $U(\Sigma y_i) + X$, where $X$ is the amount of a numéraire commodity. Total welfare is defined as the sum of consumer and producer surpluses; that is,

$$W = U(\Sigma y_i) - \Sigma_i C^i(y_i, t_i) + \Sigma_i G^i(y_i, t_i).$$

Adopting the standard convention, we take it that tax revenue is distributed to (or collected from, if the tax revenue turns out to be negative) the representative household in a lump-sum fashion.
Totally differentiating (17), making use of the utility maximization condition \( P = \frac{\partial U}{\partial (\Sigma_i y_i)} \), yields

\[
dW = \left\{ \frac{(P - C^1_y)\beta_1(1 + \alpha_2)}{\Delta} - \frac{(P - C^2_y)\beta_1\alpha_2}{\Delta} \right\} dt_1
+ \left\{ \frac{(P - C^2_y)\beta_2(1 + \alpha_1)}{\Delta} - \frac{(P - C^1_y)\beta_2\alpha_1}{\Delta} \right\} dt_2, \tag{18}\]

or, more compactly,

\[
dW = \Sigma_i \beta_i[(P - C^1_{yi}) + \alpha_j(C^1_{yi} - C^1_{yi})]dt_1/\Delta. \tag{18'}\]

Expression (18) is fully consonant with extant results for homogeneous oligopoly. Specifically, if the firms are identical and subsidies/taxes are non-discriminatory, the first-best optimum can be achieved by subsidizing them, so that \( P = C_y \). Here they are not identical, but if it were possible to tax/subsidize them at different rates, then the first-best optimum could still be achieved by setting firm-specific subsidies to ensure \( P = C^1_y = C^2_y \). It is worth pointing out that it is the possibility of demand and cost non-linearities that accounts for the presence of \( \alpha_j \) in (18') and the following results. If demand is linear, marginal costs are constant, and firms are Cournot, then \( \alpha_j \) is simply unity.

More surprising results emerge once we step outside the first-best world. Letting \( dt_1 = 0 \) in (18'), we obtain

**Proposition 3.** A tax cut extended to only firm \( f \) will be welfare-enhancing if and only if its initial mark-up is greater than the \( \alpha_j \)-weighted interfirm marginal cost differential; that is,

\[
P - C^f_y > \alpha_j(C^f_y - C^f_{yi}) \tag{19}\]

Thus, it is possible that an increase in subsidy (tax) extended only to the less (more) efficient firm would have an adverse effect on welfare if the difference in efficiency is not sufficiently large. The intuition is that a subsidy on a duopolist has two effects: (a) it increases output (the social valuation of this additional output being given by the LHS of (19)), and (b) it causes output to be redistributed in the industry towards this firm and away from the other firm. Whether such a redistribution of output is welfare-enhancing depends on the firms’ marginal costs. Also, invoking the dependence of \( \alpha_j \) on \( \epsilon \) and \( E \), it can be readily demonstrated that the more elastic and/or convex the demand, the higher the likelihood that the inequality (19) will hold.

This possibility is shown in figure 1, where \( W_i \)'s are the isowelfare contours and \( W_m \) marks the first-best subsidy pairs. If initially there were no taxes/subsidies

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17 I owe this observation to a referee.
(point 0), a subsidy given only to firm 2 will make the society worse off. This is analogous to the phenomenon uncovered by Lahiri and Ono (1988), who showed that an improvement in the technology employed by the smaller, less efficient firm could be welfare worsening.

Another common scenario is one in which institutional constraints require both firms be subject to identical changes in subsidy rates.

**Proposition 4.** A uniform tax cut will increase welfare if and only if the sum of the β-weighted mark-ups is smaller than the γ-weighted interfirm marginal cost differential; that is (recalling that $\gamma_f = -\gamma_i$):

$$\Sigma_i \beta_i (P - C^i_y) < \gamma_f (C^f_y - C^i_y)$$  (20)
We are now in a position to examine the simultaneous effects of taxes on total tax revenue, social welfare, and industry concentration. Figure 1 illustrates a typical situation. As already noted, $W_s$ are the isowelfare contours. $G_s$ are the isorevenue contours, with $G_m$ the revenue-maximizing tax pair. $SS'$ is the isoconcentration curve, here drawn for the initial condition that $t_1 = t_2 = 0$. It can be readily recognized that $W_m G_m$ is the locus of the Ramsey tax pairs: taxes that maximize welfare subject to a revenue constraint (with point $R$ in the figure corresponding to a balanced budget). The equation for this locus (omitted here) can be found by equating the slope of the isorevenue contours obtained from $(14')$ with the slope of the isowelfare contours given by $(18')$.  

Several observations can be made in connection with figure 1. First, leaving aside the exceptional case where point $R$ coincides with the origin, the balanced-budget Ramsey tax pairs would require cross-subsidization. It is entirely possible, moreover, that it is the smaller firm that would be taxed to subsidize its bigger rival. Second, since $G_m$ could be on either side of $SS'$ (see the numerical example given above), it is not possible to predict how a revenue-maximizing tax would affect concentration without additional knowledge of the industry in question. Third, $SS'$ and $W_m G_m$ may or may not intersect. If they do (as drawn), revenue- and welfare-maximizing taxes will have opposite effects on industry concentration, and further, concentration-neutral Ramsey taxes will be feasible. If they do not, both types of taxes will cause the industry to become either more concentrated or less concentrated, but it is not possible to ascertain a priori which of these outcomes will actually obtain; in addition, concentration-neutral Ramsey taxes will not be feasible in this case. This result leads us to conclude that, when the oligopoly is heterogeneous, industry concentration (as measured by the Hirschman-Herfindahl index) does not reliably mirror social welfare. An intuitive explanation is as follows: If one firm is so much more efficient than the other, it merits quasi-natural monopoly rights which, predictably, increase concentration. Viewed another way, welfare maximization might necessitate a compromise between efficiency (giving the more efficient firm a larger share of the market) and ‘fairness’ towards firms (equalizing their market power). The theory of natural monopoly is but an extreme

18 The slope of this curve is $\beta_1(s_2 + \alpha_2)/\beta_2(s_1 + \alpha_1) > 0$.
19 The Ramsey tax rates can be more directly calculated by solving the problem:

\[ \max_{t_1, t_2} W = U(\Sigma \gamma_i) - \Sigma C^i(y_i, t_i) \]

s.t. \( \dot{G} = \Sigma t_i \gamma_i \),

which yields

\[ \frac{P - C_i^f}{t_f} = \lambda \left[ 1 + \frac{1}{e_{gf}} \frac{t_f e_{gf}}{t_f + e_{gf}} \right] \frac{P - C_i^f}{t_f} s_{gf} e_{gf} \]

where \( \lambda \) is the Lagrange multiplier, \( e_{gf} \equiv (\partial y_f / \partial t_f)(t_f / t_f) = \beta_f t_f / (1 + \alpha_f) y_f < 0 \) and \( e_{gf} \equiv (\partial y_f / \partial t_f)(t_f / y_f) = e_{gf} = \beta_f t_f / \alpha_f y_f = C_i^f t_f / (t_f + y_f k_i p''') y_f < 0 \). This expression can be usefully compared with the Ramsey formula for dependent demands, separable costs, as in Tirole (1988, 70).
manifestation of this trade-off. Indeed, the first-best outcome for the preceding numerical example is a natural monopoly granted to the low-cost firm 1, raising the Hirschman-Herfindahl index to its maximum of 10,000.

IV. SUMMARY AND CONCLUSIONS

The main purpose of this paper has been to examine the effects of a firm-specific tax on a heterogeneous oligopoly. Four types of effects were analysed: on profits, on industry concentration, on total tax revenue, and on social welfare. We replicated the well-known result that taxes might benefit all firms by helping to bring about a more nearly joint profit maximization outcome. We discovered that a revenue-maximizing tax configuration could make the industry less concentrated. More surprisingly, we found that a socially optimal tax configuration could increase the concentration of the industry. The last result casts doubt on the possibility of drawing unambiguous conclusions from time-series analyses with regard to the relationship between changing industry concentration and changing social welfare. As our derivation has been based on the premise that concentration is measured by the Hirschman-Herfindahl index, it should be of interest to find out whether the same ambiguity would persist if a different index is used.

Our work can also be placed within the context of the optimal pricing problem à la Ramsey, in particular when the industry exhibits interfirm rivalry (Braeutigam 1979, 1984). Viewed from this literature, our analysis underscored the dependence of optimal policy on (i) the strategic interaction between firms and (i) the difference in firm sizes, even in the absence of economies of scale and the break-even constraint.

Finally, and at the risk of being mildly trivial, it is worth pointing out that since the optimal tax structure for a heterogeneous oligopoly would, in all likelihood, call for firm-specific tax rates, it is almost certain that a uniform tax rate (while superficially ‘fair’) would be suboptimal, regardless of the purpose of the tax.

REFERENCES