(10) 1. A metal plate is situated in the \(xy\)-plane and occupies the rectangle \(R\) \((0 \leq x \leq 8, 0 \leq y \leq 8)\), where \(x\) and \(y\) are measured in meters. The temperature at the point \((x,y)\) in the plane is \(T(x,y)\), where \(T\) is measured in degrees Celsius. Temperatures at equally spaced points were measured and recorded in the table.

\[
\begin{array}{cccccc}
   y & 0 & 2 & 4 & 6 & 8 \\
   x &  &  &  &  &  \\
   0 & 30 & 38 & 45 & 51 & 55 \\
   2 & 52 & 56 & 60 & 62 & 61 \\
   4 & 78 & 74 & 72 & 68 & 66 \\
   6 & 98 & 87 & 80 & 75 & 71 \\
   8 & 96 & 90 & 86 & 80 & 75 \\
\end{array}
\]

Estimate the value of the integral \(\iint_{R} T(x,y) \, dA\), using the Midpoint Rule with \(m = n = 2\). Be sure to give the correct units for your answer.

(20) 2. Evaluate the integral \(\iint_{D} y \, dA\) if \(D\) is the region bounded by \(y = x\) and \(x = 2 - y^2\).

(20) 3. Find the center of mass of the portion of the unit circle \(x^2 + y^2 = 1\) that lies between angles of \(-\frac{\pi}{4}\) and \(+\frac{\pi}{4}\). Assume the density is constant; call it \(\delta\) and use it in your calculation. Discuss the effect of symmetry on the location of the center of mass.

(20) 4. Set up, but don’t evaluate, an integral for the surface area of the parametric surface given by the vector function \(\mathbf{r}(u,v) = v^2\mathbf{i} - uv\mathbf{j} + u^2\mathbf{k}\), \(0 \leq u \leq 3\), \(-3 \leq v \leq 3\).

(20) 5. Consider the iterated triple integral \(\int_{0}^{1} \int_{y}^{1} \int_{0}^{y} f(x,y,z) \, dz \, dx \, dy\). Rewrite this integral with two other orders of integration, \(dx \, dy \, dz\) and \(dy \, dz \, dx\).

(20) 6. Find the moment of inertia of a solid sphere about a diameter, if the sphere is of uniform density with radius \(R\) and mass \(M\). Your answer should be expressed in terms of \(R\) and \(M\).

Hints: Choose the diameter to lie on the \(z\)-axis. Use \(\delta\) to represent the density. At the end of the problem, substitute the value of \(\delta\) in terms of \(R\) and \(M\). You may need the formula

\[
\int \sin^3(u) \, du = -\frac{1}{3} (2 + \sin^2(u))\cos(u) + C
\]