Let $c_n$ denote the $n^{th}$ Catalan number: $c_0 = 1$ and

$$c_{n+1} = \sum_{i=0}^{n} c_i c_{n-i}$$

1. Calculate $c_1, \ldots, c_7$. We know that $c_n$ equals the number of ways to triangulate a labeled, regular, $(n + 2)$-gon by vertex chords and we know that $c_4 = 14$. Up to rotational symmetry, triangulations of an unlabeled hexagon are of the types shown where the multiplicities represent the number of labeled triangulations of each type. These multiplicities add to $c_4 = 14$.

\[
\begin{array}{cc}
\text{×6} & \text{×3} \\
\text{×3} & \text{×2}
\end{array}
\]

Find all of the types and multiplicities of triangulations of the 7-gon. These multiplicities must add to $c_5 = 42$.

2. The points $1, \ldots, 2n$ are evenly spaced along a circle. Pairs of points will be linked together using $n$ non-crossing chords inside the circle (a chord may connect consecutive points along the circle as it lies within the circle). Using induction show that $c_n$ is equal to the number of ways to connect up these points with non-crossing chords.