Review of models and methodology for scheduling problems in USPS mail processing and distribution centres

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Abstract: This paper gives a comprehensive presentation of the scheduling problems to the management equipment and workforce in the United State Postal Service (USPS) mail processing and distribution centres in an effort to cut cost and increase efficiency as well as detailed discussions on some of the kernel models. These research studies advance the research frontier and have brought millions of dollars in savings to the USPS. The study of these models not only presents a successful application of operations research in the public service area, but also offers much technical and managerial insight that could be beneficial to other industries.

Keywords: postal operations; optimisation; equipment/staff scheduling; overtime management; integer program.

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1 Introduction

The United State Postal Service (USPS) is in the business of delivering mail to every household in the USA. In the year 2004, USPS delivers 206 billions of mail pieces to more than 142 million homes and businesses. The success of this large operation relies on a large network of approximately 275 major behind-the-scene mail processing and distribution centres that serve as the interfaces between local post offices and the rest of the nation.

These facilities run 24 hours a day, 7 days a week and operate a complicated manufacturing system – disassemble mail arrivals, sort and dispatch them to other facilities. To ensure timely processing, the facilities constrain a large variety of advanced equipment in the form of optical character readers, automated facer cancellers and barcode sorters for automated mail processing and employ a non-homogeneous workforce composed of full-time, part-time and casual employees that work on shifts with various lengths and start times. The scheduling of equipment (the determination of the configuration and usage of equipment to match mail arrivals) and the scheduling of workforce (the determination of the optimal size and composition of the workforce, their days off/lunch assignments, and overtime usage) to meet processing service commitment with a constantly changing demand are some of the most challenging problems.

While postal operations has attracted quite some attention research in as early as the 1960s, see Oliver and Samuel (1962), McManus (1977) and Showalter et al. (1977) for some queuing and simulation studies for the configuration and flow in these facilities; most research studies on the solution the above postal equipment and staff scheduling problems were proposed in the past ten years. In the most general sense, each scheduling problem can be decomposed temporally and hierarchical analytic approaches have been adopted. Along the time axis, these studies can be classified into strategic, tactical and operational levels. These studies advance the frontier of various operations research areas including facility design, equipment selection, workforce composition, equipment and staff scheduling, and disruptions management. Most of these models have been implemented and have brought millions of dollars of savings to the USPS.

This paper presents a comprehensive review on these research studies including detailed discussions on some kernel models with some of the latest results. The study of these models not only presents a successful application of operations research and management science in the public service area, but also offers technical and managerial insight that could be beneficial to other industries.

The remainder of the paper is organised as follows. Section 2 provides an overview of these models and the relationships among them. The focus is the tactical and operational models currently being deployed. The tactical staff scheduling model is introduced in Section 3 and the corresponding equipment scheduling model in Section 4. This is followed by the operational models where the equipment scheduling under disruption model is presented in Section 5 and the staff scheduling model under disruption in Section 6. Concluding remarks are given in Section 7.

2 Overview of the research problems and models

The postal equipment and workforce scheduling problems are some of the most challenging problems seen in industry. To better understand the complexity of the problems, several terms used to describe the characteristics of the facilities are first defined.

Mail arrival profile. Mail arrives throughout the day and an arrival profile stipulates the amount of mail received during a specific time of the day and its characteristics. The arrivals follow a highly fluctuating pattern that varies throughout the day and over the week and the total volume on a day could be anywhere from 3 to 5 million pieces.

Operation, equipment and equipment scheduling. Depending on a letter's characteristics, upon arrival, it may require several operations before it is finally dispatched. An operation is usually performed with a piece of equipment; the equipment, however, is capable of process several operations. Equipment configuration and scheduling determines the optimal size, make, and use of equipment to ensure the prompt processing of mail with the least labour cost.

Shifts and shift scheduling. To operate these machines to match the fluctuating demand, a non-homogeneous workforce is employed; each could work on many of the possible shifts with various lengths and start times. Shift scheduling finds the optimal crew size and their assignments to satisfy demand in each time period of the day.

Days off and its assignment. To construct an employee's week schedule, it is necessary to specify the days off and as such, sufficient slack must be provided through the week so that the days-off requirement is satisfied for every worker. Typically, two consecutive days off is preferable to an employee, but there is no strict restriction for it.

Break and its assignment. A lunch break is required for all shifts that exceed a certain length. For the USPS, the practice is to create a break window – a set of consecutive periods for every shift during which a break can be given. Because an employee is off the clock, there should be sufficient resources to cover for him.

Staff scheduling. The staff scheduling problem determines the optimal size and composition of the workforce and their assignments to make sure that the demand (determined by machine activities) in each time period of the week is satisfied. In the most comprehensive form, staff scheduling includes shift scheduling, days-off assignments and break assignments.

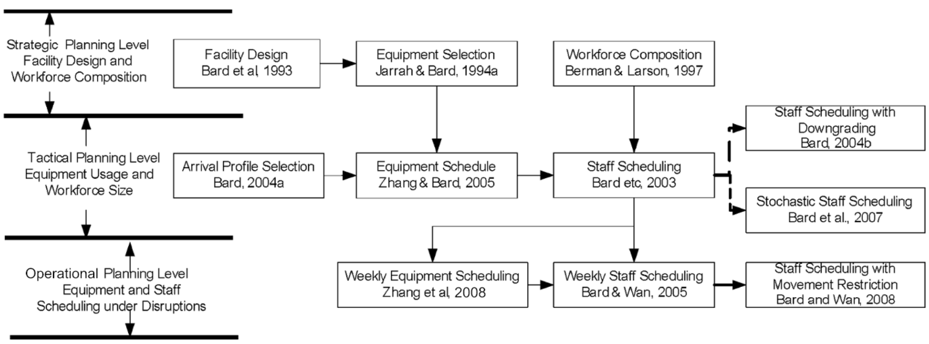
Disruptions (I) – demand fluctuation. A major disruption that affects the equipment and workforce schedule is departure from normal demand. Historic data show that dramatic seasonal and daily variations in the total amount of mail exist; Though forecasting techniques could be used to estimate the arrival, the actual arrival could differ significantly from hour to hour.

Disruptions (II) – absenteeism. Another major disruption that affects the equipment and workforce schedule is employee absenteeism. For the USPS, absenteeism could vary anywhere between 6% and 21% and significantly reduce the size of the workforce.

Schedule adjustment and overtime management. To handle disruptions, equipment schedules have to be adjusted to ensure processing commitments and additional labour resources such as called-in workers and overtime, have be resorted to complement the workforce schedule. Overtime management is used to optimally assign overtime to employees while observing contractual and union rules to match adjusted processing activities.

With these terms defined, the various models developed can be examined. Along temporal lines, these studies can be classified into three levels: strategic planning, tactical planning and operational planning. The models are listed below under each level and the relationships among them are shown in Figure 1. In the figure, the blocks represent the models; the solid arcs represent the dependence relations and the dashed arcs extension relations. The studies on the equipment scheduling are lined up on the left and the studies on the staff scheduling on the right. The various models are listed under each level as follows.

Figure 1 Relationships among the models under discussion



Strategic planning level. The problem at this level is how to design a facility, to determine the capacity and makes of the equipment, and to evaluate the impact of various workforce policies.

Facility design. Bard et al. (1993) studied the facility design problem and presented a two-level approach that started with a large-scale mixed integer program. The solution was then input to a simulation model that was used to investigate operational issues related to service standards, growth in mail volume, and the use of new equipment.

Equipment selection. Jarrah et al. (1994a) studied the equipment selection problem. The problem was to make a choice among multiple machine makes. A mixed integer linear program was first solved to select equipment and propose a tentative schedule.

This was followed by a linear program to compress or eliminate the idle time of the machines.

Workforce composition. Berman et al. (1997) studied the workflow management and workforce composition problem. The problem was modelled as a queuing network and linear programming was used in the analysis. This study examined various policies such as full time-to-part time ratio, the switching of jobs during a day, et cetera.

Tactical planning level. The problem at this level is how to generate an equipment schedule that matches mail arrivals and to determine the optimal size and composition of a permanent workforce. Supplement issues include how to select the best arrival profile to use in these analyses and how to address uncertainty in the demand while configuring the workforce.

Equipment scheduling. Zhang and Bard (2005) studied the equipment scheduling problem. The problem took as the input the *average* arrival profile, whose selection was proposed in the arrival profile selection model (Bard, 2004a) and tried to determine the optimal use of equipment. The problem was modelled as an integer program and a surrogate shift covering constraints was used to capture labour costs to provide a link to combine equipment with workforce scheduling.

Staff scheduling. Jarrah et al. (1994b) were the first to study staff scheduling problem. The problem was modelled as an integer program that combined shift scheduling and days-off scheduling in a unified manner. These ideas were expanded in Bard et al. (2003) with several new features incorporated. The staff scheduling problem was to find the optimal size and composition of a permanent workforce to meet the demand (generated from the equipment scheduling) and was the kernel of tactical planning.

Arrival profile selection. Bard (2004a) studied the selection of the best arrival profile when running the staff scheduling model. Because demand varied throughout the year, the choice of the input data was crucial. If a week of low volume was selected, the solution might call forth an insufficient number of workers; if a week of high volume is chosen, excessive idle time might result. The selection of the best 'average' arrival profiles was solved using an efficient trial and error approach to find the lowest volume whose slack is sufficient to cover all weeks of greater volume without exceeding the guidelines for use of overtime, part timers and casuals.

Staff scheduling with down grading. Bard (2004b) extended the staff schedule problem to a multi-skilled workforce. Demand was specified by skill type, and in the downgrading analysis, a person in a higher skill category could be assigned a job in a lower skill category, but at the higher rate of pay. A mixed-integer linear programming model for this problem was developed based on staffing requirements.

Stochastic staff scheduling. Bard et al. (2007) extended the staff scheduling problem that took demand uncertainty into consideration. They proposed a two-stage stochastic integer program with recourse for the analysis. In the first stage, before the demand was known, the number of employees was determined. In the second stage, demand was revealed and workers were assigned to specific shifts over the week. When necessary, overtime and casual workers s were used to satisfy the demand.

Operational planning level. The problem at this level is how to adjust the equipment and employee schedules to meet the service commitment in the face of various disruptions such as demand from normal and employee absenteeism.

Staff scheduling under disruptions. Bard and Wan (2005) studied the weekly staff scheduling problem under disruptions. The problem here was how to adjust employee schedules by overtime assignment, slight modification of employee configuration such as increasing the number of part-time hours, and calling in temporary workers in response to an updated demand (generated from equipment schedule under disruption).

Equipment scheduling under disruptions. Zhang et al. (2008) studied the equipment scheduling under disruptions and proposed an integer program for the analysis. The model took as input the forecast arrival profile and workforce and attempted to make the optimal adjustments to equipment and staff schedules to meet processing commitment. The model was solved for each day of the coming week. Several analyses were conducted to evaluate the effects of the use of overtime and the controlling of absenteeism.

Staff scheduling with movement restriction. Wan and Bard (2007), and Bard and Wan (2008) later extended the weekly staff scheduling problem under disruptions to a multi-skilled workforce when movement restrictions exist between workstation groups. A new model is proposed that integrates WSG restrictions with the shift scheduling and task assignment constraints. The model takes the form of a large-scale integer program and is solved with one of two decomposition heuristics.

Other models. Several other studies have also been proposed. Particularly, Judice et al. (2004) proposed an integer program for a lot-sizing and workforce problem in these facilities. Wang et al. (2005) studied sequencing the processing of incoming mail in order to match a given outbound truck delivery schedule. Qi and Bard (2006) proposed a simulation and optimisation techniques to generate the staff requirements.

In the following sections, we give more detailed descriptions of the four kernel models in this framework, namely, the equipment scheduling model by Zhang and Bard (2005), the staff scheduling model by Bard et al. (2003), the equipment schedule model under disruption by Zhang et al. (2008), and the weekly workforce scheduling model by Bard and Wan (2005), that are currently implemented in the USPS and have brought millions of savings to the USPS. The focus here is to present the various aspects of postal staff scheduling problems as well as the role equipment scheduling plays in the management of demand for staff scheduling models. Much technical and managerial insight is presented with latest results on significant cost reduction, especially when equipment and staff scheduling are modelled in an integrated manner.

3 The staff scheduling model

The staff scheduling problem

- finds the optimal size and composition of a permanent workforce
- constructs weekly tours for all employees that comply with union and contractual rules to satisfy a given demand.

The earlier works by Berman and Larson (1993) and Berman et al. (1997) on workforce configuration and overtime management only served as policy analysis at a strategic level, and are not sufficient for tactical planning purposes. Malhotra and Ritzman (1994) and Malhotra et al. (1992) are among the first to consider the various issues faced by the postal service in scheduling a flexible workforce. The postal staff scheduling model presented here was first studied by Jarrah et al. (1994b) and later finalised by Bard et al. (2003).

3.1 The baseline model for staff scheduling

The whole problem is divided into three components where the first component is the shift scheduling. The shift scheduling problem begins with the definition of all the possible shifts for both part-time and full-time employees and concludes with the number of employees that should be assigned to each shift to satisfy daily demand. The second component of the weekly schedule requires the specification of days off. The optimal workforce size should be provided with sufficient slack throughout the week to satisfy the days off requirement for each employee. The last component of the weekly schedule is to accommodate lunch breaks, assigned to be within a specific break window for each shift.

For the USPS, a full time worker works 8½ consecutive hours, which include a half-hour lunch break. A part-timer, on the other hand, may be assigned to one of the many possible-length shifts. All employees working more than 6 hours per day must be given a ½ hour lunch break. The breaks are to be assigned between the 9th and the 12th periods. Each worker must be given two days off, preferably two consecutive days or at least one Saturday or Sunday off, per week

Given these requirements, the baseline staff scheduling model staff can be modelled as an integer program as follows. The following notation is used in the development of model.

Indices

d: Index for the days of the week; $d = 1, \ldots, 7$

t: Index for time periods during a day; $t = 1, \ldots, 48$

f: Index for the full-time shift types; $f = 1, \ldots, n^F$

p: Index for the part-time shift types; $p = 1, \ldots, n^P$.

Parameters

c_f: Prorated weekly cost of full-time shift f

c_p: Prorated weekly cost of part-time shift p

G_{ft}: 1 if full-time shift type f covers period t; 0 otherwise

P_{pt}: 1 if part-time shift type p covers period t; 0 otherwise

D_{dt}: Demand for period t on day d

n^F: Number of full-time shifts

n^P: Number of part-time shifts

ρ: Full-time to part-time labour ratio.

Decision variables

x_{fd}: Number of employees assigned to full-time shift type f on day d

y_{pd}: Number of employees assigned to part-time shift type p on day d

β_{dt}: Total number of breaks in period t on day d

w_f: Total number of full-time employees needed for shift type f

v_p: Total number of part-time employees needed for shift type p.

The staff scheduling model

$$\text{Minimise } z = \sum_{f=1}^{n^F} c_f w_f + \sum_{p=1}^{n^P} c_p v_p \tag{1}$$

subject to

$$\sum_{f=1}^{n^F} G_{ft} x_{fd} + \sum_{p=1}^{n^P} P_{pt} y_{pd} - \beta_{dt} \geq D_{dt}, \quad d = 1, \ldots, 7; t = 1, \ldots, 48 \tag{2}$$

$$\sum_{f=1}^{n^F} w_f \geq \rho \sum_{p=1}^{n^P} v_P \tag{3}$$

$$w_f \geq \frac{1}{5} \sum_{d=1}^{7} x_{fd}, \quad f = 1, \ldots, n^F \tag{4}$$

$$w_f \geq x_{fd}, \quad f = 1, \ldots, n^F; d = 1, \ldots, 7 \tag{5}$$

$$v_p \geq \frac{1}{5} \sum_{d=1}^{7} y_{pd}, \quad p = 1, \ldots, n^P \tag{6}$$

$$v_p \geq y_{pd}, \quad p = 1, \ldots, n^P; d = 1, \ldots, 7 \tag{7}$$

$$w_f \geq 0, v_p \geq 0, \beta_{dt} \geq 0, x_{fd} \geq 0, y_{pd} \geq 0 \text{ and integer}, \quad \forall t, k, p, d \tag{8}$$

β_{dt}: modelled through implicit modelling of breaks.

The objective function (1) minimises the total weekly cost of the workforce. Constraint (2), assures that the net workforce is sufficient to cover the demand. The net workforce is the total number of part-time and full-time employees whose shift definitions cover that specific period, less those who have a break during that period. The latter is modelled using a methodology proposed by Bechtold and Jacobs (1990) (for details, please see the Appendix). The 0–1 matrices (G and P) filter out shifts that do not cover the period under consideration. Constraint (3) limits the number of part-time employees.

Constraints (4)–(7) are used to calculate lower bounds on the number of workers required to meet the daily demand. The first of these bounds, $L1$, is needed to assure that there is enough coverage so that every worker can take two days off per week. Constraints (4) and (6) correspond to $L1$, which equals $(1/5)\sum_{d=1}^{7} x_{fd}$ for full-timers and $(1/5)\sum_{d=1}^{7} y_{fd}$ for part-timers, respectively. The second lower bound, $L2$, is necessary to assure that a sufficient number of workers exist to cover the day with the highest demand. Constraints (5) and (7) correspond to $L2$, which equals max$\{x_{fd}$: $d = 1,\ldots,7\}$ for full-timers and max$\{y_{fd}$: $d = 1,\ldots,7\}$ for part-timers, respectively.

3.2 Extensions to enforce two consecutive days off

The most significant extension to the baseline model is the addition of the requirement that an employee have two consecutive days off in a week. Alfares (1997) used a simple approach that provided an exact lower bound, W, on the size of the workforce that allows two consecutive days off for each worker. Alfares further proved that for a (5,7)-cyclic scheduling problem with consecutive days off requirements, the lower bound for shift type f can be calculated as follows.

$$w_f = \max\left\{(\max_d x_{fd}), \left\lceil \frac{1}{5}\sum_{d=1}^{7} x_{fd} \right\rceil, \left\lceil \frac{R_{\max}}{3} \right\rceil \right\} \tag{9}$$

where

$$R_i = \sum_{d \in S_i^*} x_{fd}, \quad i = 1,\ldots,7 \quad \text{and} \quad R_{\max} = \max_i R_i$$

and S_i^* is the complement of the set of three nonadjacent numbers given in Table 1.

Table 1 Set of three nonadjacent numbers and its complement

I	S_i	S_i^*
1	{3,5,7}	{1,2,4,6}
2	{1,4,6}	{2,3,5,7}
3	{2,5,7}	{1,3,4,6}
4	{1,3,6}	{2,4,5,7}
5	{2,4,7}	{1,3,5,6}
6	{1,3,5}	{2,4,6,7}
7	{2,4,6}	{1,3,5,7}

The first two terms in equation (9) are the first and second lower bounds (4)–(7) in the baseline model. As such, the addition of the following constraints will provide the third lower bound and guarantee a schedule in which every employee can be given two consecutive days off.

$$w_f \geq \frac{1}{3}\sum_{d \in S_i^*} x_{fd}, \quad f = 1,\ldots,n^F, i = 1,\ldots,7 \tag{10}$$

$$v_p \geq \frac{1}{3}\sum_{d \in S_i^*} y_{pd}, \quad p = 1,\ldots,n^P, i = 1,\ldots,7. \tag{11}$$

3.3 Solution framework and results

The staff scheduling model is currently being implemented nationwide and changed the way the facility manages its workforce. The model has also been adopted for various staffing policies analysis. To illustrate, consider the following two policies: the first policy gives two days off, while the second gives two consecutive days off. Table 2 shows the costs of these two different days off policies.

Table 2 Staffing results for different days off policies

	Total cost per week	*Number of full-timers*	*Number of part-timers*	*% consecutive days off*
Any two days off	\$96,280	101	25	68.9
Consecutive days off	\$103,600	108	27	100
Increase	7.7%	–	–	–

If an employee is given any two days off, rather than two consecutive days off, a dramatic decrease, 7.7%, in the cost can be observed. Though giving two consecutive days off is preferable to employees, this result suggests a heavy price must be paid if such a policy is adopted. While this is not surprising, many organisations still follow such costly practice. For USPS, the staff scheduling model provides a tool to perform these analyses and points out ways to reduce staff budget. However, to reflect employee preference, the staff scheduling model has attempted to assign as many employees two consecutive days off as possible. The percentages of workers with two consecutive days off under the two policies are reported under the '% consecutive days off' column in the table.

4 The equipment scheduling model

Though the staff scheduling problem has received much attention in the literature, the solution of the equipment scheduling problem was not systematically studied until recently. The earlier models on the equipment side, the facility design model by Cebry et al. (1992), Bard et al. (1993) and Jarrah et al. (1994a), did not directly address the interaction with staff scheduling. In these models, staff requirements were mostly of minor importance and were roughly estimated. However, this is not the case for tactical equipment scheduling where the purpose is to provide an equipment schedule for use with staff scheduling. A new model has to be developed and this model is fully studied in Zhang and Bard (2005).

4.1 The multi-level lot sizing model with shift covering constraints

Upon arrival, a letter undergoes several operations before it is dispatched early the next day. For letter processing, this procedure is done in three major steps:

- cancelling the stamp if one exits
- reading the destination address and identify it with a barcode
- sorting the mail to its final destination.

The processing activities can best be modelled as a multi-level lot sizing problem with additional shift covering constraints to capture the labour cost.

The full mathematical model is presented below and the following notation is used in the development of the model.

Indices

i, o: Indices for input and output mail streams

p, n: Indices for operations

m: Index for machine groups

t: Index for time periods; $t \in T = \{1, \ldots, 48\}$

f: Indices for shifts.

Sets

I, O: Input and output mail streams

M, $M(n)$: All machine groups and machine groups capable of performing operation n

N, $N(m)$: All operations and operations performed by machine group m

$P(n)$: Operations immediately preceding operation n

$I(n)$, $O(n)$: Input mail streams to and output mail stream from operation n

$T(n)$: Periods during which operation n can be performed

$T(i)$, $T(o)$: Periods in which input mail i is accepted or output stream o is processed

F: Shifts.

Parameters

$a_i(t)$: Amount of external mail of stream i arriving in period t; $t \in T(i)$

$q_m(t)$: Number of machines available in group m in period t

ρ_n: Processing rate for operation n

f_{pn}: Fraction of mail processed at predecessor operation p that is sent to operation n

τ_1, τ_2: Time required to start up or clear a machine

h_{ft}: Whether period t lies within the start and end periods of shift f or not

r_m: Number of employee required to run machine m.

Decision variables

$v_n(t)$: Inventory level of operation n at the end of period t

$w_{mn}(t)$: Amount of mail processed for operation n by machine group m in period t; $t \in T(n)$

$Y_{mn}(t)$: Number of machines devoted to operation n by machine group m in period t; $t \in T(n)$

$Z^1_{mn}(t)$: Number of startups at the beginning of period t; $t \in T(n)$

$Z^2_{mn}(t)$: Number of clearances at the end of period t; $t \in T(n)$

ω_f: The number of workers assigned to shift f.

The mathematical model for equipment scheduling problem is as follows.

$$v_n(t-1) + \sum_{i\in I(n),t\in T(i)} a_i(t) + \sum_{m\in M(p)} \sum_{p\in P(n)} f_{pn} w_{mp}(t-1) - \sum_{m\in M(n)} w_{mn}(t) = v_n(t) \quad \forall n \in N, t \in T \tag{12}$$

$$w_{mn}(t) + \frac{\tau_1}{30}\rho_n Z^1_{mn}(t) + \frac{\tau_2}{30}\rho_n Z^2_{mn}(t) \le \rho_n Y_{mn}(t) \quad \forall m \in M(n), t \in T(n), n \in N \tag{13}$$

$$Z^1_{mn}(t) - Z^2_{mn}(t-1) = Y_{mn}(t) - Y_{mn}(t-1) \quad \forall t \in T, m \in M, n \in N \tag{14}$$

$$\sum_{n\in N(m)} Y_{mn}(t) \le q_m(t) \quad \forall t \in T, m \in M \tag{15}$$

$$\sum_{n\in N(m)} \sum_{m\in M(n)} r_m Y_{mn}(t) \le \sum_{f\in F} h_{ft}\omega_f \quad \forall t \in T \tag{16}$$

$$Y_{mn}(t), \omega_f \text{ integer} \quad \forall t \in T(n), m \in M(n), n \in N, f \in F. \tag{17}$$

Here, constraint (12) states that the quantity of mail at the beginning of a period minus those pieces processed during the period should equal the ending inventory. Constraint (13) ensures that the workload devoted to mail processing plus the lost capacity due to startups and clearances should not exceed a machine's production capacity. Constraint (14) defines the start-up and clearance activities and constraint (15) stipulates that at any time period, the machines in operation cannot be more than total machines available. Constraint (16), which is referred to as *the shift covering constraints* ensures that the number of workers on duty in any time period should be sufficient to match the number requested to operate the machines.

Constraints (12)–(15), in essence, define a multi-level lot sizing model that keeps track of mail processing and machine activities throughout the day. The novelty of the model is the introduction of the shift covering constraint (16). The addition of this constraint captures the staffing cost in an equipment schedule and provides a link between equipment and staff scheduling models.

4.2 Solution framework

Several criteria can be identified in the evaluation of an equipment schedule. The first criterion is to process all the mail arrivals during the day and can be achieved by minimising the ending inventory at the end of the day as follows.

$$\text{Minimise} \sum_{n\in N} v_n(48).$$

The second criterion is to minimise the number of shifts or workers to process the mail and can be stated as follows.

$$\text{Minimise} \sum_{k\in K} \sum_{f\in F} \omega_{kf}.$$

This number of full time shifts represents a surrogate for the daily staffing cost and is essential, as we will see later, to the quality of solution obtained from staff scheduling.

To further refine the equipment schedule, a third criterion, to minimise a combination of the number of startups and the weighted sum of working periods, is proposed.

$$\text{Minimise} \sum_{n\in N}\sum_{m\in M(n)}\sum_{t\in T} Z^{1}_{mn}(t) + \sum_{n\in N}\sum_{m\in M(n)}\sum_{t\in T}(1-0.01\,t)\,Y_{mn}(t).$$

The first term is the total number of startups. Minimising the number of startups reduces the lost capacity in which the machines are switched on and off. The second term is a weighted sum of working periods, which, when being minimised, will essentially push an operation to as late in the operation window as possible. This implicitly serves to shorten the working intervals of an operation, a quality preferred by facility managers.

The problem is essentially a multi-criteria mixed integer program and is solved using a pre-emptive approach. To improve computation time, Zhang and Bard (2006) developed a LP-based three phase heuristic to solve some of the difficult models. The heuristic uses a LP solution as a target and tries to find integer solutions as close as possible to the LP and was able to find solutions orders of magnitude faster than the standard branch ad bound algorithm.

4.3 The impact of using of equipment schedule as a demand management tool

Equipment scheduling serves as a front-end to staff scheduling and, as we will see, is critical to the final solution obtained from the staff scheduling model. To demonstrate the importance of equipment scheduling, two significant results are summarised below.

4.3.1 The use of shift covering constraint as the surrogate for staffing cost

The key component in the equipment scheduling model is the use of a surrogate for labour costs through the use of shift covering constraints. To see the effect of adding this constraint, Table 3 reports the optimal staffing costs as well as the numbers of full-timer and part-timer imposed by equipment schedules that are obtained with and without the shift covering constraints.

Table 3 Costs from staff scheduling with and without the shift covering constraint

Performance measure	*Weekly cost*	*Full-timers*	*Part-timers*
Without shift covering constraint	\$135,928	108	27
With shift covering constraint	\$127,018	101	25
Reduction	6.4%	–	–

As we can see, a reduction of 6.4% in staffing costs was achieved from schedule generated with shift covering constraints compared to the one without shift covering constraints. Because all parameters were set the same, this difference was caused only by the demand of workers imposed by the equipment schedule. This result clearly demonstrates the importance of the shift covering constraint and allows the equipment and staff scheduling to be solved in an integrated manner.

4.3.2 The existence of equipment schedule as a demand management tool

The importance of equipment scheduling in the framework can be demonstrated by comparing the staffing costs associated with a schedule generated from the proposed optimisation approach and one generated from the rule of thumb from the facilities. To show this, the staffing costs, as well as the numbers of full-time and part-time workers in these two cases, are reported in Table 4.

Table 4 Costs from optimisation and from rule of thumb

Performance measure	*Weekly cost*	*Full-timer*	*Part-timer*
Rule of thumb	\$208,221	162	40
Optimisation approach	\$144,544	112	28
Reduction	44%	–	–

As we can see, a striking reduction of 44% in staffing costs can be achieved from the equipment schedule generated through the optimisation approach compared to the rule of thumb schedule. From another angle, equipment scheduling serves as a demand management tool that smoothes the demand of workers to staff scheduling and leads to better staff solutions. The optimisation of equipment scheduling provides a most direct way to achieve cost reduction. If only 50% of the benefits are realised, roughly \$1.6 million in savings per facility can be achieved.

The arrival profile in the equipment scheduling model represents the average values, as does the demand for workers used in the staff scheduling model. In realty, the arrival profiles could change dramatically from month to month and from week to week, thus render the optimal schedule from the tactical models infeasible, let along optimal. Re-planning is necessary and this calls for solution of the operational scheduling problems – the equipment scheduling problem under disruptions and the workforce scheduling problem under disruptions.

5 The equipment scheduling model under disruption

The equipment scheduling model under disruption (Zhang et al., 2008) was developed to schedule the equipment based on forecast demand and employee absenteeism. The solution to this problem requires an even more intimate optimisation of equipment and workforce because lunch breaks and overtime assignment, which can be ignored in the tactical equipment scheduling model, have to be considered here in a short term operational planning model.

5.1 Mathematical model

The mathematical model of the equipment scheduling under disruption problem is composed of three modules. Each module deals with a different aspect of the problem. The first one is the equipment scheduling or lot sizing module that keeps track of the mail processing and machine activities. The second one is the shift scheduling and overtime management module that addresses the assignment of overtime to employees, and the third one is the break assignment module that accommodates lunch breaks to be assigned within a break window.

Besides the notation used in the development of the equipment scheduling model, the following additional notation is used.

Indices

s: Indices for shifts (with overtime).

Sets

$S(f)$: Set of shifts where employee working on shift f can take (with overtime).

Parameter

n_f: Number of employees scheduled to work on shift f on a day.

Decision variables

x_s: Number of employees assigned to overtime shift s, $s \in S(f), f \in F$

ω_s: Number of casual workers to work on shift s, $s \in S(f), f \in F$

β_t: Number of breaks in period t.

The full mathematical model for the equipment scheduling model under disruption is presented below.

Equipment scheduling module: Constraints (12)–(15).

Shift scheduling and overtime management module:

$$\sum_{n \in N(m)} \sum_{m \in M(n)} r_m Y_{mn}(t) \le \sum_{f \in F} \sum_{s \in S(f)} h_{st} x_s + \sum_{f \in F} \sum_{s \in S(f)} h_{st} \omega_s - \beta_t \quad \forall t \in T \tag{18}$$

$$\sum_{s \in S(f)} x_s = n_f \quad \forall f \in F. \tag{19}$$

Break assignment module: β_t is modelled here.

Constraint (18) states that the total number of active shifts should be adequate to cover the requirement of workers to run the machines. It is essentially the shift covering constraint defined earlier, but with the inclusion of shifts for part timers and called-in casual workers, and with the deduction of number of breaks initiated in the period. Constraint (19) states that exactly n_f employees, who were assigned to shift f, can be assigned to the overtime shifts associated with shift f. In addition, the following constraint is necessary to model the fact that the total overtime cannot exceed a certain percentage, currently set at 6%, of the total work hours.

$$\sum_{f \in F} \sum_{s \in S(f)} o_{fs} x_{fs} \le 0.06 \sum_{f \in F} l_f n_f. \tag{20}$$

Here, l_f the length of a standard full time shift and o_{fs} is the number of overtime periods in overtime shift s with respect to standard shift f, computed as $o_{fs} = l_s - l_f$, where l_s is the length of the overtime shift s; $s \in S(f)$ and l_f is the length of the original shift f.

5.2 Solution framework and results

The objectives here are:

- to minimise the ending inventory
- to minimise the overtime and casual costs
- to minimise a combination of the number of startups and the weighted sum of working periods.

While the first and the third objective are defined earlier, the second objective can be written as follows

$$\sum_{f \in F} \sum_{s \in S(f)} c_s x_s + \sum_{f \in F} \sum_{s \in S(f)} \overline{c}_s w_s \omega_s$$

where c_s is the overtime cost of extending an employee's standard shift to overtime shift s, and $\overline{c}_s$ is the cost of a call-in casual that works on shift s. The first term represents the overtime costs while the second term represents the cost of called-in casuals. The model is solved using a preemptive approach similar to the tactical equipment scheduling, and therefore is not presented.

Several experiments have been conducted to gain any managerial insights and some of the most significant results are summarised here.

Absenteeism. A basic question is how much absenteeism can affect the overtime and casual costs. Table 5 reports these costs under different absenteeism ratios, 0, 3, 6, 9, 12, 15, 18, and 21%. The percentage increase row reports the percentage increase of overtime costs under the current ratio against that under the previous one.

Table 5 Weekly and annual overtime costs

Ratio	0%	3%	6%	9%	12%	15%	18%	21%
Cost (year)	29,777	48,874	65,486	86,807	118,397	136,203	168,931	187,481
% Increase	–	64.1%	34.0%	32.6%	36.4%	15.0%	24.0%	10.1%

The existence of absenteeism destroys the optimal staff schedule and as such, overtime and casuals have to be used to complement a worker's schedule and to maintain the workforce size. This is not surprising; however, this result suggests that a significant increase in overtime cost occurs when absenteeism increases. For example, when the absenteeism increases from 6% to 21%, the costs almost triples from $65,486 to $187,481. This suggests research on policies and studies to lower the absenteeism could lead to tens of millions of financial savings for USPS.

Overtime usage. The impact of overtime usage is shown in Table 6 where the overtime costs under various overtime ratios are presented. Here, the absenteeism ratio is fixed at 12%.

Table 6 Cost under overtime ratio

Overtime ratio	0%	3%	6%	9%
Annual cost	182,247	124,907	124,801	123,241
Decrease	–	31.5%	0.08%	1.25%

This result clearly demonstrates the effectiveness of overtime. When the overtime ratio is increased from 0% to 3%, cost reduction of 31.5% is observed. The use of overtime increases worker efficiency, eliminate unnecessary called-in workers, and could sharply reduce the costs incurred in the face of uncertainty and disruptions, as observed by Easton and Goodale (2005).

The equipment scheduling under disruption model is solved for each day of the week and, as such, cannot model overtime requirements such as the total number of over time hours or days that are imposed on a weekly basis. These issues are being taken into consideration in the weekly staff scheduling model under disruption.

6 The weekly staff schedule model under disruption

The weekly staff scheduling problem under disruption is studied by Bard and Wan (2005). This model is designed to make weekly assignments and overtime assignment to the staff schedule based on a forecast demand for the coming week.

The following notation is used in the development of the model. For conciseness, notation inherited from the staff scheduling model such as indices d to represent the days of the week, t to represent time period of a day, and demand D_{dt} for each period in a day are omitted.

Indices

k: Index for full-time and part-time regular employees, $k \in K^R$

s: Index for shifts associated with the full timer or part timers $s \in S(k, d)$; for casuals, $s = 1, \ldots, n^C$.

Parameters

C_{kds}: Cost when regular employee k works shift s on day d

C_{ds}: Cost of part-time shift s on day d

$\hat{c}_k$: Penalty overtime hourly rate for regular employee k

C_s: Cost of casual shift type s

H_{kdst}: 1 if shift s on day d covers period t for employee k; 0 otherwise

P_{kst} : 1 if part-time shift type$s$ covers period t for employee k; 0 otherwise

C_{st} : 1 if casual shift type s covers period t; 0 otherwise

l_s : Length of part-time shift s

o_{ks} : Amount of overtime associated with shift s for employee k

$PD_{\min}$: Minimum number of days per week that must be assigned to a PTF (=2)

PD_{max}: Maximum number of days per week that can be assigned to a PTF (=6)

PH_{min}: Minimum number of hours per week that must be assigned to a PTF (=15)

PH_{max}: Maximum number of hours per week that can be assigned to a PTF (=39).

Sets

K^R: Set of full-time and part-time regular employees

K^L: Set of part-time flexible employees

$W(k)$: Set of days employee k is scheduled to work as defined by his or her bid job

$\bar{W}(k)$: Set of days employee k is off (complement of $W(k)$)

$S(k, d)$: Set of shifts that employee k is permitted to work on day d

$\hat{S}(k,d)$: Set of overtime shifts that employee k is permitted to work on day d.

Decision variables

x_{kds} : (Binary) 1 if regular employee k works shift s on day d; 0 otherwise

y_{kds} : (Binary) 1 if part-time flexible employee k is assigned to shift p on day d; 0 otherwise

v_{ds} : Number of casual shifts of type s required on day d

β_{dt} : Total number of breaks initiated in period t on day d

γ_{kd} : Amount of penalty overtime (double time) worked by employee k on day off d

δ_{kd} : (Binary) 1 if d is the first off day for employee k; 0 if d is the second day off.

The mathematical model for the staff scheduling model under disruption is presented below.

$$\text{Minimise } z = \sum_{d=1}^{7}\sum_{k\in K^R}\sum_{s\in S(k,d)} c_{kds}x_{kds} + \sum_{d=1}^{7}\sum_{k\in K^L}\sum_{s\in S(k,d)} c_{ds}y_{kds} + \sum_{d=1}^{7}\sum_{s=1}^{n^C} c_s v_{ds} + \sum_{d=1}^{7}\sum_{k\in \bar{W}(d)} \hat{c}_k \gamma_{kd} \quad (21)$$

subject to

$$\sum_{k\in K^R}\sum_{s\in S(k,d-1)} H_{k,d-1,s,t+48}x_{k,d-1,s} + \sum_{k\in K^R}\sum_{s\in S(k,d)} H_{kdst}x_{kds} + \sum_{k\in K^L}\sum_{s\in S(k,d-1)} P_{ks,t+48}y_{k,d-1,s} + \sum_{k\in K^L}\sum_{s\in S(k,d)} P_{kst}y_{kds} + \sum_{s=1}^{n^C} C_{s,t+48}v_{d-1,s} + \sum_{s=1}^{n^C} C_{st}v_{ds} - \beta_{d-1,t+48} - \beta_{dt} \geq D_{dt}, \quad d=1,\ldots,7;\ t=1,\ldots,48 \quad (22)$$

$$\sum_{s\in S(k,d)} x_{kds} = 1, \quad k\in K^R;\ d\in W(k) \quad (23)$$

$$\sum_{s\in S(k,d)} y_{kds} \le 1, \quad k \in K^L; d = 1,\dots,7 \tag{24}$$

$$PD_{\min} \le \sum_{d=1}^{7} \sum_{s\in S(k,d)} y_{kds} \le PD_{\max}, \quad k \in K^L \tag{25}$$

$$PH_{\min} \le \sum_{d=1}^{7} \sum_{s\in S(k,d)} l_s y_{kds} \le PH_{\max}, \quad k \in K^L \tag{26}$$

$$x_{kds}, y_{kds}, \delta_{kd} \in \{0,1\}, v_{ds}, \gamma_{kd} \ge 0 \text{ and integer}, \beta_{dt} \ge 0, \quad \forall k,d,s,t. \tag{27}$$

The objective function (21) minimises the total weekly cost of the workforce. Constraint (22) is the demand satisfaction constraint. The number of breaks initiated in period t, β_{dt}, is defined in the break assignment module (see Appendix), and thus is not repeated. Constraints (23) and (24) regulate the working hours for full-timers and part-timers respectively. Constraints (25) and (26) limit the weekly schedule for part-time workers to six days and 39 h respectively. The weekly overtime usage restrictions are modelled through constraints given below.

$$\sum_{d=1}^{7} \sum_{s\in \hat{S}(k,d)} o_{ks} x_{kds} \le OT_{\max}, \quad k \in K^R \tag{28}$$

$$\sum_{d\in W(k)} \sum_{s\in \hat{S}(k,d)} x_{kds} \le OD_{\max}, \quad k \in K^R \tag{29}$$

$$\sum_{k\in K^R} \sum_{d=1}^{7} \sum_{s\in \hat{S}(k,d)} o_{ks} x_{kds} \le 0.06 \left(\sum_{k\in K^R} \sum_{d=1}^{7} \sum_{s\in S(k,d)} l_s x_{kds} + \sum_{k\in K^L} \sum_{d=1}^{7} \sum_{s\in S(k,d)} l_s y_{kds} \right) \tag{30}$$

$$\sum_{s\in \hat{S}(k,d)} o_{ks} x_{kds} \le \gamma_{kd} + 8\delta_{kd}, \quad d \in \overline{W}(k), k \in K^R \tag{31}$$

$$\sum_{d\in \overline{W}(k)} \delta_{kd} \le 1, \quad k \in K^R \tag{32}$$

where $OT_{\max}$ is the maximum number of overtime hours permitted in a week and is set to 20. $OD_{\max}$ is the maximum number of scheduled days of overtime and is set to four. Constraints (28) and (29) set upper bounds on overtime hours and overtime days, respectively. Constraint (30) describes the restriction with respect to overtime ratio while constraints (31) and (32) determine the order of the two days off for the sake of minimising penalty overtime.

6.2 Relationship between operational equipment and staff scheduling models

To run the weekly staff scheduling model under disruption, ideally, a demand of workers should be provided though a demand management tool; this is what exactly the equipment schedule under disruption is designed for. To show this, the costs from the weekly staff scheduling model and that from tactical equipment scheduling are reported in Table 7.

Table 7 Comparison of staffing costs from weekly staff scheduling

Absenteeism ratio	6%	12%	18%
Equipment schedule	122,172	191,485	241,457
Equipment schedule under disruption	65,486	118,397	168,931
Reduction (%)	46.4%	38.2%	30.0%

As we can see, significant reductions in overtime and casual costs, as much as 30–50%, can be achieved using the schedules generated from the equipment scheduling model under disruption, compared to these generated from the tactical equipment schedule under normal demand. This clearly demonstrates the necessity of the equipment schedule under disruption model, which provides an accurate adjustment equipment schedule and serve as the demand tool for use with the weekly staff scheduling model for overtime adjustments in the coming week.

The weekly staff scheduling model, on the other hand, produces the weekly schedule of workers, the input to the equipment scheduling model under disruption. Unlike other disruption management models, for USPS, the staff scheduling model under disruption and the equipment scheduling model under disruptions have to be modelled with a holistic view. These two models share a close relationship, complement each other, and jointly accomplish the management of resources in these facilities on a routine basis.

7 Conclusions

This paper provides a comprehensive review of the scheduling issues and problems in USPS P&DCs and research studies in the past ten years to address them. These studies start with strategic models to find the optimal workforce configuration and to determine the different types of equipment needed in a facility. This is followed by the tactical models to determine the optimal size and composition of a permanent workforce. Finally, operational models are developed to make optimal adjustments to equipment and workforce schedules to meet processing commitment in the face of demand fluctuation and absenteeism. These studies advance the frontier of various operations research areas; present one of the most successful applications of operations research in public service industry and have brought millions of dollars of financial savings for the USPS. While posts everywhere are under pressure to improve delivery service, reduce operating expense, and make better use of their large capital investment, these operations research models and decision support tools could help dramatically increase operation efficiency and reduce labour cost – the USPS has reported a labour reduction as much as 20~30% and throughput increase of 30~50% and millions of dollars of financial saving.

These models are mostly large-scale integer optimisation programs and managerial insights have been gained though various parameter and policy analyses. To push the frontier even further, it is necessary to take demand uncertainty and absenteeism uncertainty into consideration while building mathematic models. Though there have been some studies that addressed these problems, the still lack stochastic optimisation models partly because of the lack of data and partly because of the difficulty in the solution of these large stochastic models. Improved equipment and personnel schedule models could lead to significant reductions in labour costs and remains to be a challenge yet to be solved.

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References

Alfares, H.K. (1997) 'An efficient two-phase algorithm for cyclic days-off scheduling', *Computers and Operations Research*, Vol. 25, No. 11, pp.913–923.

Bard, J.F. (2004a) 'Selecting the appropriate input data set when configuring a permanent workforce', *Computers and Industrial Engineering*, Vol. 47, No. 4, pp.371–389.

Bard, J.F. (2004b) 'Staff scheduling in high volume service facilities with downgrading', *IIE Transactions on Scheduling and Logistics*, Vol. 36, No. 10, pp.985–997.

Bard, J.F. and Wan, L. (2005) 'Weekly scheduling in the service industry: an application to mail processing & distribution centers', *IIE Transactions on Scheduling and Logistics*, Vol. 37, No. 5, pp.379–396.

Bard, J.F. and Wan, L. (2008) 'Workforce design with movement restrictions between workstation groups', *Manufacturing and Service Operations Management*, Vol. 10, No. 1, pp.24–42.

Bard, J.F., deSilva, A.H., Feo, T.A. and Wert, S.D. (1993) 'Design of semi-automated mail processing facilities', *IIE Transaction on Design and Manufacturing*, Vol. 25, No. 4, pp.88–101.

Bard, J.F., Bicini, C. and deSilva, A.H. (2003) 'Staff scheduling at the United States Postal Service', *Computer and Operational Research*, Vol. 30, pp.745–741.

Bard, J.F., Morton, D.P. and Wang, Y.M. (2007) 'Workforce planning at USPS mail processing & distribution centers using stochastic optimization', *Annals of Operations Research*, Vol. 155, No. 1, pp.51–78.

Bechtold, S.E. and Jacobs, L.W. (1990) 'Implicit modeling of flexible break assignments in optimal shift scheduling', *Management Science*, Vol. 36, No. 11, pp.1339–1351.

Berman, O. and Larson, R.C. (1993) 'Optimal workforce configuration incorporating absenteeism and daily workload variability', *Socio-Economic Planning Sciences*, Vol. 27, No. 2, pp.91–96.

Berman, O., Larson, R.C. and Pinker, E. (1997) 'Scheduling workforce and workflow in a high volume factory', *Management Science*, Vol. 43, No. 2, pp.158–172.

Cebry, M.E., deSilva, A.H. and Dilisio, F.J. (1992) 'Management science in automating postal operations: facility and equipment planning in the United States Postal Service', *Interfaces*, Vol. 22, No. 1, pp.110–130.

Easton, F.F. and Goodale, J.C. (2005) 'Schedule recovery: unplanned absences in service operations', *Decision Sciences*, Vol. 36, No. 3, pp.459–488.

Jarrah, A.I.Z., Bard, J.F. and deSilva, A.H. (1994a) 'Equipment selection and machine scheduling in general mail facilities', *Management Science*, Vol. 40, No. 8, pp.1049–1068.

Jarrah, A.I.Z., Bard, J.F. and deSilva, A.H. (1994b) 'Solving large-scale tour scheduling problems', *Management Science*, Vol. 40, No. 9, pp.1124–1145.

Judice, J., Martins, P. and Nunes, J. (2004) 'Workforce planning in lot-sizing mail processing problem', *Computer and Operations Research*, Vol. 40, No. 9, pp.3031–3058.

Malhotra, M.K. and Ritzman, L.P. (1994) 'Scheduling flexibility in the service sector: a postal case study', *Production and Operations Management*, Vol. 3, No. 1, pp.1–17.

Malhotra, M.K., Ritzman, L.P., Benton, W.C. and Leong, G.K. (1992) 'A model for scheduling postal distribution employees', *European Journal of Operational Research*, Vol. 58, No. 3, pp.74–85.

McManus, I.M. (1977) 'Optimum use of overtime in post offices', *Computers and Operations Research*, Vol. 4, No. 4, pp.271–278.

Oliver, R.M. and Samuel, A.H. (1962) 'Reducing letter delays in post offices', *Operations Research*, Vol. 10, No. 6, pp.839–892.

Qi, X. and Bard, J.F. (2006) 'Generating labor requirements and rosters for mail handlers using simulation and optimization', *Computers and Operations Research*, Vol. 33, No. 9, pp.2645–2666.

Showalter, M.J., Krajewski, L.J. and Ritzman, L.P. (1977) 'Manpower allocation in US postal facilities: a heuristic approach', *Computers and Operations Research*, Vol. 4, No. 4, pp.271–278.

Wan, L. and Bard, J.F. (2007) 'Weekly scheduling with workgroup restrictions', *Journal of the Operational Research Society*, Vol. 58, pp.1030–1046.

Wang, Q., Betta, R. and Szczerba, R. (2005) 'Sequencing the processing of incoming mail to match an outbound truck delivery schedule', *Computers and Operations Research*, Vol. 32, No. 7, pp.1777–1791.

Zhang, X. and Bard, J.F. (2005) 'Equipment scheduling at mail processing and distribution centers', *IIE Transactions on Scheduling and Logistics*, Vol. 37, No. 2, pp.175–187.

Zhang, X. and Bard, J.F. (2006) 'Comparative approaches to equipment scheduling in high volume factories', *Computer and Operations Research*, Vol. 37, No. 1, pp.132–157.

Zhang, X., Chakravarthy, A. and Gu, Q. (2008) 'Equipment scheduling under disruption at USPS mail processing and distribution center', *Journal of Operational Research Society*, to appear.

Appendix: Modelling of the total number of breaks

The total number of breaks initiated in a period t, denoted as β_t, for a set of shifts, each has its break window, is implicitly modelled using a methodology proposed by Bechtold and Jacobs (1990). In the development of the model, the following notation is used.

Indices

s, t: Time period.

Parameters

k, q: The earliest and latest periods a break can begin for any of the permissible shifts.

Sets

F: Set of shift types that have breaks

B_s: $\{j$: break window for shift j lies entirely between period s and $q\}$

F_s: $\{j$: break window for shift j lies entirely between periods k and $s\}$

M, N Sets of initial and final periods of the break windows, both in ascending order.

The constraints to model the break initiated in period t are as follows.

$$\sum_{t=k}^{s} \beta_t - \sum_{f \in F_s} x_f \geq 0, \forall s \in N \tag{33}$$

$$\sum_{t=s}^{q} \beta_t - \sum_{f \in B_s} x_f \geq 0, \forall s \in M \tag{34}$$

$$\sum_{f \in F} x_f - \sum_{t=k}^{q} \beta_t = 0. \tag{35}$$

The first constraint, (33), is referred to as the *forward pass* constraint. It assures that the total number of breaks initiated form period k up to a given period s exceeds the total number of employees who should have taken their breaks by that period. The employees included in the constraint are those whose break windows are fully covered through s, but not the ones who have the option of a break in some future period. The second constraint (34) is referred to as the *backward pass* constraint and assures that the total number of breaks that are initiated from some specific period s through the end of the day (or until the last period that can be taken as a break, which is q) exceeds the number of employees who are entitled to a break during this interval. The last constraint (35), known as the balance equation, is needed to assure that every worker is assigned a break and that it is within its permitted window.