

# Opti-Money at Bank Hapoalim: A Model-Based Investment Decision-Support System for Individual Customers

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Opti-Money is a decision-support system for allocating assets that was developed at Bank Hapoalim, Israel's largest bank. Based on customer- and market-specific inputs, we solve a modified Markowitz-type nonlinear programming problem to produce optimal tailor-made investment portfolios in terms of asset classes. In its five years of operation, this mature system has provided excellent quantifiable results for the bank and its customers. In 2002, we held some 133,000 consultation sessions with 63,000 customers in which we used Opti-Money. Apart from the considerable prestige the system gained through its track record and uniqueness in the Israeli banking sector, it obtained net income 88 percent higher in customer accounts that used Opti-Money than in those that did not. The annual earnings over and above benchmarks to customers who follow the investment advice provided by the system total US\$244 million. In 2002, the annual income for the bank directly attributed to Opti-Money exceeded US\$31 million.

*Key words:* financial institutions: banks; decision analysis: applications.

The Bank Hapoalim Group is Israel's largest banking group and the leading force in virtually every area of banking and financial activity. As a universal bank, established in 1921, the bank provides a wide range of banking and financial services and products to retail, private banking, commercial, and large corporate customers in Israel and abroad. In Israel, the group provides these services through a network of 327 branches and nine regional business centers and through the offices of domestic subsidiaries, including four commercial banking subsidiaries, financial companies for investment banking, mortgages, credit cards, mutual and provident funds, trust services, and portfolio management. Worldwide the group operates through 37 branches, offices, and subsidiaries in major financial centers in North and South America and Europe, focusing on trade financing, corporate and syndicated financing, and private banking. The group also owns interests in a number of nonfinancial companies in insurance, tourism, and real estate. As of December 31, 2002 the group had US\$55.4 billion

in assets and US\$2.8 billion in shareholders' equity. Domestically, the bank has a market share of over 30 percent; globally, Bank Hapoalim ranks as No. 127 in the world. The bank is rated A2 by Moody's and BBB+ by Fitch.

## Background Information

In 1995, several factors converged that caused us to conceive the Opti-Money system. First, Bank Hapoalim, facing increased competition in the domestic market, embarked on a major restructuring program based upon a customer-segmentation strategy and a realignment of its divisions according to customer types. As part of this restructuring, it established private banking units that were to develop the most innovative methodology and technology for investment advice, a service unavailable at competitor banks. The customer relations managers (CRM), highly trained bank officers and investment advisors, give customers personal and individual solutions to

their banking needs. In particular, CRMs help private banking customers plan their investments. The CRMs are part of the bank's securities and financial assets division, which is responsible for giving customers investment advice, for doing research and trading securities in Israel and abroad, and for developing and producing new banking products and managing existing products. While the CRMs followed the principles of diversification and selected portfolios to reflect customers' risk attitudes, they had no quantitative models or tools available for providing systematic advice.

Second, in the same year, the Investment Advice and Investment Portfolio Management Law, which regulates CRMs' advisory activities, was enacted in Israel. The law was intended to provide a legislative framework for regulating investment advisors and portfolio managers. It explicitly defines advisory activity, the licensing of persons permitted to give advice, the obligations imposed on advisors, those entitled to receive advice, and the requirements for documenting consultation sessions. Paragraph 12 of the law, which concerns adapting services to customers' needs, states that

A license holder will match, as far as possible, the advice that he gives to his customers or the nature of the transactions that he conducts on their behalf to the needs and guidelines of each customer, after verifying with the customer the investment objective, and his financial position....

Third, as part of its restructuring, the bank decided to upgrade the financial products it offered to customers and to improve its management of risk exposure. To accomplish these tasks, the bank needed to use analytic and computational methods. It established an operations-research-oriented department for analytic development. The new department was to find and identify finance areas that could benefit from quantitative methods, to develop mathematical models and computational tools, and to implement them for users. We built the Opti-Money system with systems-oriented support from the operations division.

## Initial Considerations

—The system should be suitable for thousands of customers with various investment profiles (for example, different objectives, horizons, risk appetites, and tax statuses). Therefore, its portfolio recommendations must be tailor-made for every customer. The system was intended primarily for customers of the private banking units, who are high-net-worth customers and who expect personal attention.

—Whereas the bank can ask customers about most aspects of their profiles, it must assess their appetites for risk indirectly. Consequently, it needed a questionnaire that would elicit an indication about customers' attitudes towards risk.

—We decided that the system should not make buy and sell recommendations for customers. We wanted to preserve the advisory aspect of the CRMs' work. We decided that the system should construct portfolios at the level of general asset class, such as stock and bond groups and banking-product groups. Then, the CRMs would still be responsible for providing the detailed composition within the asset classes, thereby retaining their investment preferences. The bank strongly believes in such a combination of formal and creative approaches to investment practice.

—CRMs usually give customers investment advice in face-to-face consultations in the bank's branches or by phone. At the end of a consultation session, the CRM and the customer should know the precise composition of the portfolio and the buy and sell orders needed to modify an existing portfolio. In response to input from the CRM, the system should provide an instant solution so that the CRM can base specific recommendations on the recommended asset allocation to guide the customer.

—We thought that the optimization model would probably be a nonlinear programming problem and that it would be impractical for CRMs to cope with difficulties in obtaining solutions. Any such difficulties would undermine the customers' trust in the advisory process and could result in loss of business. Consequently, we decided to enumerate all possible combinations of customer inputs and process all the optimization runs periodically in a batch (usually at the beginning of each month) and store the results in a central location. CRMs then retrieve the solutions relevant to particular customers during consultation sessions. The organizational restructuring required the purchase of new computer hardware for the CRMs in the branches, which could be less expensive with one proposed framework for the optimization runs than it would have been if the CRMs had run the optimizations.

—Because the batch runs would consist of tens of thousands of optimizations, we took special measures with regard to problem formulation and coding to ensure their successful completion in an unattended (overnight) mode.

## Conceptual Implementation

In early 1996, the bank assembled a development team consisting of representatives of the securities and financial assets division and the department of

analytic development. The team members from the securities and financial assets division described a typical consultation session. They saw the agenda of such a session as follows: The CRM first explains Opti-Money to the customer and its part in developing an investment portfolio tailored to the customer's needs and preferences. Second, because CRMs must be trained and government-licensed to use the Opti-Money system and customers must sign consultation agreements to obtain advice with the help of the system, both must sign in. Next, the CRM inputs the customer's data, needs, and preferences into a user-friendly Microsoft Windows-based interface.

The interface displays four screens of questions.

(1) The first, the investment preference screen, concerns three parameters: The customer's investment objectives, horizon, and benchmark.

—The customer can choose among such objectives as investment for a personal event, investment for buying a house, investment for saving on behalf of children, or investment with no specific objective.

—The customer can choose an investment horizon, such as one year, two years, five years, or 10 years.

—For a benchmark, the customer chooses an index to exceed. Four benchmarks are currently available in the system: The consumer price index (CPI) (inflation), the riskless rate of interest (the Bank of Israel's "bellwether" interest rate), the US dollar exchange rate, and the Euro exchange rate. For example, real estate prices in Israel are quoted in US dollar terms. The appropriate index for a young couple saving for their first house would therefore be the US dollar benchmark. On the other hand, for a person saving for retirement, the CPI benchmark would be most suitable. The benchmark selected affects the output of the Opti-Money system in two ways: The first effect is in the selection of assets; for example, the system recommends a higher ratio of CPI-indexed assets if the customer chooses the CPI benchmark. The second effect is that the assessment of asset risk is based on the historical performance of the chosen benchmark. Customers can choose multiple investment objectives and benchmarks, and the system's results will be weighted accordingly.

(2) The second screen displays the bank's forecast of two main macroeconomic indices affecting the Israeli capital market, the CPI and the rate of change in the US dollar exchange rate. The customer or the CRM has the option of overriding the bank's forecast with his or her own forecast of these indices.

(3) The third screen concerns the customer's preference with regard to liquidity and currency. Because the universe of asset classes contains both liquid investments (for example, bonds and stocks) and illiquid ones (for example, certificates of deposit (CDs)),

the customer can set limits on the two types of investment in the optimal portfolio. The customer should also specify the currency in which the investment is to be made (Israeli versus foreign).

(4) The fourth screen concerns the customer's risk-taking classification. The system assigns each customer to one of five levels: low-risk level, low-medium-risk level, medium-risk level, risk-oriented level, and high-risk level. The system assigns customers to these levels based on their answers to the questionnaire on this screen.

After the input phase, the system broadcasts the customer's profile to a server that holds the optimal portfolios of every possible profile. The server instantly returns the sought output. The Opti-Money output screen first displays the optimal portfolio, in tabular and graphical form, in terms of major asset classes: equities (domestic and foreign), foreign currency fixed-income assets; domestic CPI-linked fixed-income assets; and domestic unlinked fixed-income assets. This display shows the big picture of the portfolio. Next, the system breaks down the major asset classes into asset classes whose fractions in the portfolio are the variables in the optimization model.

If the customer is not satisfied with the portfolio the model recommends, the CRM can go back to the input screens, change some parameters, and obtain a new portfolio composition. Once the customer is satisfied with the portfolio composition, the CRM recommends individual assets in each asset class to buy or sell. The investment horizon of a recommended portfolio is one year, that is, investors usually restructure their portfolios once a year. The bank, however, urges its customers to come in for checkup consultation sessions every six months, during which they can adjust their portfolios with the help of Opti-Money and the CRMs.

The team members from the department of analytic development built the conceptual model, wrote the software, obtained the necessary market data and forecasts, incorporated them into the model, and transmitted the optimal solutions to the CRMs' screens. Our natural choice of modeling framework was the classical Markowitz model of mean-variance portfolio selection (Markowitz 1959). Yet, in addition to linking return to risk in general terms, investors want to track or beat benchmarks and maintain balanced portfolios. We, therefore, modified the formulation of the basic Markowitz model to suit our needs. Another important aspect of our modeling approach was that we had to construct thousands of portfolios (some 36,000) in a batch mode. This amounted to an automated industrial production of optimal portfolios based on nonlinear programming without human interaction. For the solution of the nonlinear programs, we chose the GAMS software system, with which we had previous experience, and

the actual solver we used is MINOS, which can also handle problems that include nonlinear functions with discontinuous derivatives. We wrote an extensive input-output interface in Delphi, which communicates between the GAMS/MINOS system and the data retrieval and storage facilities.

## Actual Implementation

After a development phase of about 22 months, we installed Opti-Money in 47 private banking units in early 1998. By February 2003, we had installed the system at the desks of all CRMs. Over 485 CRMs at 228 branches are now working with the help of the system. In the first month in which the system was installed, CRMs used it in only 224 consultation sessions. For the sake of comparison, CRMs used the system in 1,330 consultation sessions in December 2000 and in 17,743 consultation sessions in December 2002 (Figure 1).

To facilitate integration of the system, we conducted three training sessions for CRMs:

(1) In the initial training session, we explained how to operate the system and how the model works.

(2) In the second session, we explained how to integrate the system into consultation sessions, how to present it to customers, how to persuade customers that they need to answer the questionnaires, and how to explain when they may need the system.

(3) In the third session, we explained the system's underlying model, benchmarks, and simulations in greater detail.

Overall, we conducted some 100 group training sessions on the use of the system. In addition, we also instructed the branch managers about its use to acquaint them with the system's added value, to ensure that they would encourage use of the system, and to inform them about the managerial tools inherent in the system.

As part of integrating the system, we produced various learning aids: (1) an extensive users' guide; (2) a brief guide to the system that included explanations of the screens that facilitate use of the system; (3) a PowerPoint presentation on integrating the system into consultation sessions; (4) a brochure for customers explaining model-based computerized consultation systems; and (5) study material for

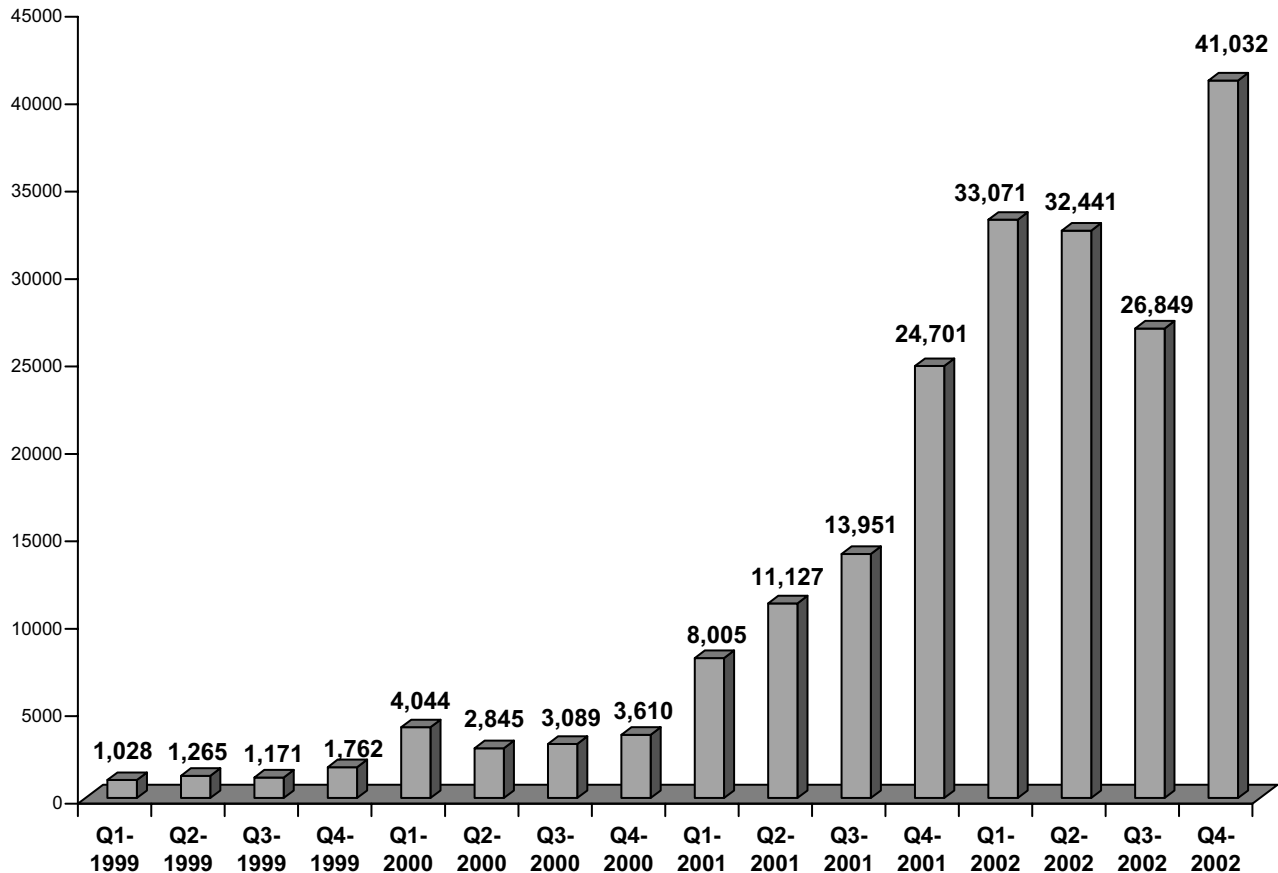


Figure 1: The quarterly usage of the Opti-Money system in consultation sessions shows a dramatic increase from a few thousand in 1999–2000 to more than 40,000 at the end of 2002.

supporting the system, which describes the improvements made to the system over the years.

Apart from conducting training sessions and developing study aids, since 1998 when we first introduced the system, we have been promoting use of the system among the CRMs. We intend this advisory initiative activity to encourage CRMs to give investment advice in general and, in particular, to integrate Opti-Money into the consultation process. At the beginning of each year (the activity is now in its sixth year), we publish an activity schedule and announce a competition among branches. This schedule of events is designed to encourage consultation activity in all its forms and use of the system for decision support. At the end of each measurement period as defined in the schedule (this year, every trimester), we measure CRM performance and remunerate outstanding CRMs and branches.

The variables of the model are the weights of the various asset classes in the investment portfolio. These asset classes are classes of stocks traded on the Tel Aviv stock market grouped by historical return, risk characteristics, and market capitalization; Israeli government bond classes grouped by their yield characteristics (for example, CPI or US dollar linked) and maturity; foreign (Euro) bond classes grouped by maturity; foreign stock indices (for example, the Standard and Poor (S&P) 500) and a class of Israeli stocks traded in the US; and classes of saving plans, deposits, and structured products offered by the bank. All together, some 60 asset classes are represented in the model.

Expected return forecasts on the asset classes and benchmarks included in the model are impor-

tant inputs. A senior advisors' forum coordinated by the securities and financial assets division provides monthly forecasts. The forum consists of advisors who serve affluent customers; equity analysts; regional advisors (who supervise the CRMs at the branches); advisors specializing in foreign currency and investments abroad; advisors specializing in banking products, such as deposits and savings plans; a representative of the bank's economics department; and senior macro analysts specializing in the Israeli capital market. The forum is convened monthly with special sessions to assess the effects of major economic or political events.

The bank estimates the direct cost of developing the system at \$220,000 for personnel and \$20,000 for software and hardware. Because of the system's importance in the consultation process and the bank's responsibility for its results, the Opti-Money system is subject to constant control, support, and maintenance activity, currently performed by a staff of three people from the securities and financial assets division and the department of analytic development.

## Track Record

Opti-Money is a mature decision-support system, serving Bank Hapoalim for the last five years. But did it live up to expectations? To answer this question, let us look at the track record of the system, as can be seen in the following figures.

We recorded the system's performance over the four-year period from January 1, 1999 to December 31, 2002 relative to the three benchmarks defined in the model (Figures 2–4): the CPI, the US dollar

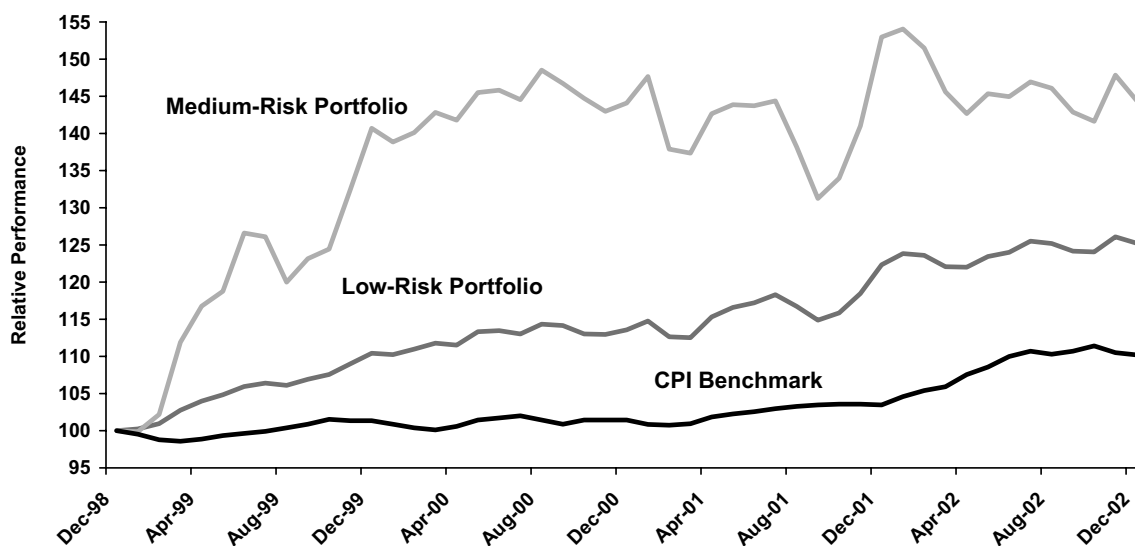


Figure 2: Over a period of four years (1999–2002), the low-risk and medium-risk CPI-based Opti-Money portfolios outperformed the CPI benchmark by 15 and 34 percents, respectively.

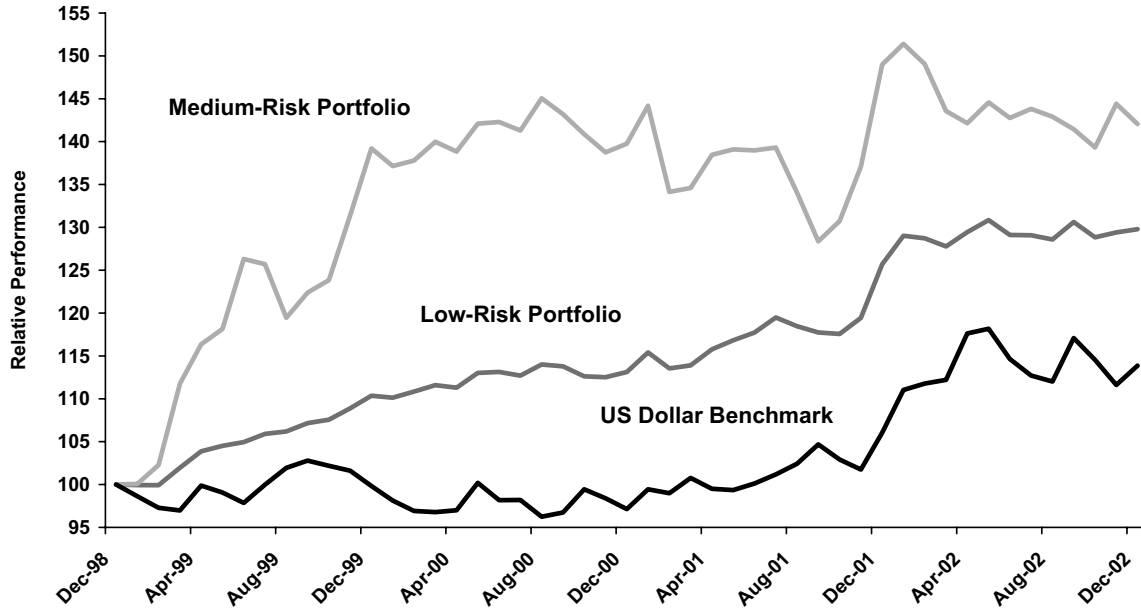


Figure 3: Over a period of four years (1999–2002), the low-risk and medium-risk US dollar-based Opti-Money portfolios outperformed the US dollar benchmark by 16 and 28 percents, respectively.

exchange rate, and the shekel represented by the Bank of Israel’s bellwether interest rate (the euro exchange rate benchmark is new and has no sufficient track record). For each benchmark, we calculated the cumulative return of two portfolios: a low-risk portfolio and a medium-risk portfolio. The characteristics of these portfolios are those that were the most pop-

ular among the investors. That is, they are for private investors, regular taxpayers with a horizon of five years, no special restrictions on liquidity, and no modifications in the bank’s macroeconomic forecasts. On a risk-scale of 1 to 5 (with 1 being most conservative), the low-risk portfolio has a risk-scale of 1 and the medium-risk portfolio a risk-scale of 3. Each

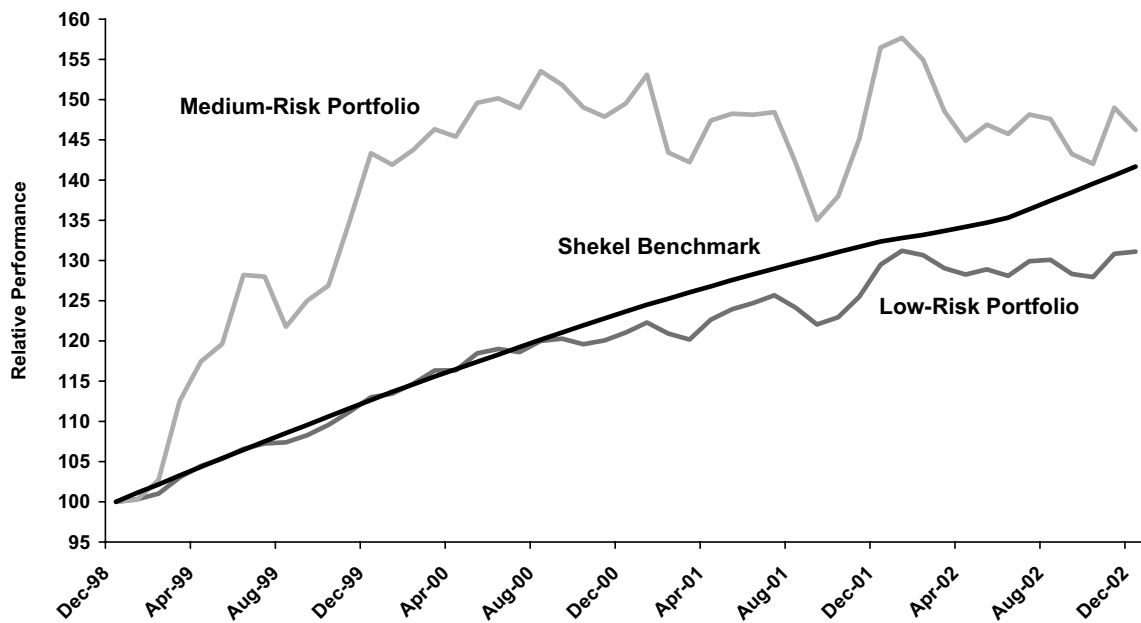


Figure 4: Over a period of four years (1999–2002), the low-risk shekel-based Opti-Money portfolio underperformed the shekel benchmark by 10.6 percent, whereas the medium-risk portfolio outperformed the same benchmark by 4.6 percent.

portfolio is restructured at the beginning of each calendar year by the then prevailing recommendations of Opti-Money.

Five of the six portfolios had greater returns than their benchmarks. The higher the risk level of the recommended portfolio, the greater the return. One portfolio of the six failed to achieve a return higher than its benchmark. This was the low-risk portfolio with the shekel benchmark that, by definition, should invest much of the portfolio in unindexed local-currency riskless assets. The shekel benchmark used in Opti-Money is the Bank of Israel's bellwether rate of interest, announced at the end of every month. Unfortunately, no completely risk-free investment products consistently yield a return higher than that rate.

We encountered some difficulties in the early phases of the Opti-Money project. The CRMs did not immediately embrace the computerized decision-support system as part of the investment advisory process. They feared that the system would replace them and that the system would reduce, if not totally nullify, their added value in the advisory process. We alleviated their fears by making it clear to them in the training sessions that the system is a decision-support tool and not a substitute for their value expertise. Some CRMs expressed apprehension regarding the model's ability to allocate assets appropriately largely because most were not familiar with the use of models in their line of work. We had to make a great effort to gain their confidence in the model's output.

Some of the older investment advisors were unhappy with the prospect of using computers in their work; some had never used personal computers. In addition to training them to use the system, we gave them intensive instruction in using a computer. But we also learned from the complaints of the users of the system. For example, some CRMs had macroeconomic forecasts that differed from those of the forecasting forum. We modified the system so that the CRMs or the customers can input their own forecasts of the major macroeconomic parameters by adjusting them up or down by 30 percent and obtain an optimal solution different from the default solution of the system. This feature enabled the CRMs and the customers to examine the effects of various macroeconomic scenarios on the output of the model and to gain confidence in the credibility of the model.

As a special service, we are working with large institutional investors to run the system in special sessions using forecasts based on their individual assessments. The system makes recommendations accurately suited to their assessments of the capital market.

Account Type	Monthly Profit
Retail accounts that do not use Opti-Money	142.5 shekels
Private banking accounts that do not use Opti-Money	224.2 shekels
Private banking accounts that use Opti-Money	422.0 shekels

**Table 1: We compared average monthly profits for retail accounts in 2002 and found that the average monthly profit the bank derives from private banking customers who use Opti-Money (422.0 shekels) is 88.2 percent higher than that for private banking customers who do not use the system (224.2 shekels) and 196.1 percent higher than that of all retail customers who do not use the system (142.5 shekels).**

## Contribution to the Bank's Profitability

We evaluated the contribution of consultations employing the Opti-Money system for 2002. We examined the contribution to the bank's profit of the retail accounts of customers who obtained advice with the use of the system, as compared to customers with similar attributes who did not receive Opti-Money advice (Table 1).

Among customers, who do not use Opti-Money, the average monthly profit per account of private banking customers (224.2 shekels) is 57.3 percent higher than the average monthly profit from all retail accounts (142.5 shekels). The average monthly profit derived from private banking customers who use Opti-Money (422.0 shekels) is 88.2 percent higher than that of private banking customers who do not use the system (224.2 shekels) and 196.1 percent higher than that of all retail customers who do not use the system (142.5 shekels).

In 2002, 63,225 customers used the system on 133,393 occasions; thus, the average number of consultation sessions with Opti-Money held in 2002 was 2.1 per customer. This number is consistent with our recommendation that customers attend consultation sessions every six months.

In examining the data we saw that 93 percent of customers who participated in consultation sessions using the system in 2002 belonged to the private banking segment. Because nearly all uses of the system were for customers in that segment, we base our profitability calculations on the assumption that CRMs used the system only for private banking customers.

Customers who attend consultation sessions using the Opti-Money system contribute an extra  $(422.0 - 224.2) = 197.8$  shekels to the bank's average monthly profit. In addition, we know that 63,225 customers participated in consultation sessions using the system in 2002. The system's contribution to the bank's profit is, therefore  $197.8 * 63,225 = 12,505,905$  shekels a month. It follows that the system's contribution to the bank's profit in a year is  $(12,505,905 * 12) =$

150,070,860 shekels. Using an average 2002 exchange rate of shekel/US\$4.74, we obtain an annual contribution to the bank's profit of  $(150,070,860/4.74) = \text{US}\$31,660,519$  for the Opti-Money system. We have little information on how the CRMs select customers to become users of Opti-Money. It may be, for example, that they prefer to introduce customers with large portfolios to the system.

## Contribution to Customers

We examined the performance of six portfolios with different risk levels (low and medium) and different benchmarks generated by the Opti-Money system on January 1, 1999. In these portfolios, we assumed that the composition (the asset classes) matched exactly the Opti-Money system's recommendations. The returns were computed for the asset classes without going into the details of the individual assets. We also assumed that the portfolios were restructured at the beginning of each calendar year by the then prevailing recommendations of Opti-Money. We examined the cumulative return on these portfolios on December 31, 2002 after four years of investments (Table 2).

We examined the earnings of the 63,225 customers who received advice using the Opti-Money system in 2002. The average size of their asset portfolios was 556,505 shekels. The total assets were thus  $63,225 * 556,505 = 35,185,028,625$  shekels. The average annual

excess return was 3.29 percent. It follows that the annual earnings attributable to Opti-Money (in excess of the various benchmarks) totaled  $35,185,028,625 * 0.0329 = 1,157,587,442$  shekels, or  $\text{US}\$244,216,760$ . We based these calculations on the assumption that those customers who use Opti-Money accept the system's recommendations fully. We compared a typical optimal portfolio recommended by Opti-Money and an average actual portfolio recommended by the CRMs in a consultation session with Opti-Money and found they differed by plus or minus three to five percent in terms of major asset classes, such as stocks or bonds. In other words, the CRMs final recommendations were quite close to those of the system.

## Additional Contributions

Bank Hapoalim's subsidiaries benefit from the system as well. Apart from serving CRMs and customers at all banking subsidiaries, the system made a major contribution to the profitability of another subsidiary. The fund managers at Continental Mutual Fund Management were impressed by the system, by its analytical ability, and by the quality of the recommended portfolios, which are based on the integration of customers' requirements and a mathematical model. Once aware of the potential of the system, they issued a mutual fund, called Optimum Solidi (Conservative Optimum), whose holdings are based on the Opti-Money recommendations for a popular type of portfolio (the portfolio of a conservative customer with a low-risk level, an unindexed shekel benchmark, and an investment horizon of two years who wished to invest only in liquid assets). Continental issued the fund on September 10, 2001 after investment advisors and customers had acknowledged the Opti-Money system as a leading decision-support system. Investors bought a large number of units in the fund as soon as it was launched. The issue of such a quantitative fund was regarded as a major event in the mutual fund market in Israel, and Continental was surprised by the large volume of units sold and by the impact on the company as a whole (Table 3). The fund's management fees from its inception to the end of 2002 totaled 17,423,000 shekels, which amounted to more than 60 percent of the total management fees collected by Continental for that period.

Finally, the success and popularity of the Opti-Money system has led to an organizational revolution in the investment advisory process at Bank Hapoalim. To assist the CRMs in recommending particular securities, we have developed additional consultation-support systems.

Portfolio Type	Portfolio Return From Jan. 1, 1999 to Dec. 31, 2002 (%)	Portfolio's Excess Return Over Benchmark (%)	Portfolio's Excess Return Over Benchmark, Annualized (%)
Low-risk portfolio with CPI benchmark	25.23	15.03	3.56
Medium-risk portfolio with CPI benchmark	44.42	34.22	7.63
Low-risk portfolio with US dollar benchmark	29.79	15.92	3.76
Medium-risk portfolio with US dollar benchmark	42.05	28.2	6.4
Low-risk portfolio with shekel benchmark	31.11	-10.57	-2.75
Medium-risk portfolio with shekel benchmark	46.24	4.56	1.12
Equally weighted average return		14.56	3.29

**Table 2:** We examined the cumulative performance of recommended portfolios from January 1, 1999 through December 31, 2002. (The data are consistent with those in Figures 2 to 4.)

	From Sept. 10, 2001 to Dec. 31, 2001	2002	Total
The fund's management fees (shekels)	1,601,000	15,822,000	17,423,000
Continental's management fees (shekels)	3,210,000	24,674,000	27,884,000
Contribution of Optimum Solidi to Continental's profitability (%)	49.88	64.1	62.5

**Table 3: By launching Optimum Solidi, Continental greatly increased its profitability.**

## Appendix: The Mathematical Model

Given  $n$  financial assets whose future returns are uncertain, we construct a portfolio of these assets to achieve some performance measure desirable to the investor. The construction of the portfolio is equivalent to determining the fractions  $x_1, x_2, \dots, x_n$  of assets  $i = 1, \dots, n$ , such that

$$\sum_{i=1}^n x_i = 1, \quad x_i \geq 0.$$

We have two types of data on the asset returns:

(1) A history of  $t = 1, 2, \dots, T$  past periodical returns,  $r_{t,i}$ , on all the assets (e.g., monthly or quarterly returns over a period of several years). Typically, we use monthly returns with  $T = 36$ .

(2) Expected future returns,  $\rho_i$ , on all the assets (e.g., for the next year).

In general, the expected future return of asset  $i$  is not the same as the expected value of the historical returns on asset  $i$ . The periodical historical return,  $\text{Portf Ret}_t$ , on a portfolio composed of these assets is the weighted (by the asset fractions) arithmetic average of the individual asset returns for that period,

$$\text{Portf Ret}_t = \sum_{i=1}^n r_{t,i} x_i, \quad t = 1, 2, \dots, T,$$

and similarly for  $\text{Exp Ret}$ , the expected future return on the portfolio

$$\text{Exp Ret} = \sum_{i=1}^n \rho_i x_i.$$

The model uses the historical returns data internally to compute risk measures. The expected return forecasts on the assets are exogenous to the optimization model. The classic approach to determining the fractions is the Markowitz-type investment model. The Markowitz model considers *portfolio variance*, a measure of a portfolio's volatility (i.e., up- and down-variability around its own average historical return value), as risk. It maximizes the expected return on

the portfolio subject to a given risk level or minimizes risk subject to a given expected return level.

The foundation on which the Opti-Money model rests is a Markowitz-type investment model, which we modified considerably both from a conceptual and a practical viewpoint. We list these modifications.

### The Concept of Risk

Most investors prefer measuring the performance of their portfolios with respect to some given target portfolio or benchmark and consider risk as the variability of their portfolio around the returns of the benchmark. Moreover, underperforming a benchmark is less desirable than outperforming it and, therefore, risk defined as downside risk (i.e., underperforming) with respect to a benchmark may be more suitable to some investors.

In Opti-Money, we defined risk in more general terms than the classical portfolio variance of Markowitz. The following risk measures are available in Opti-Money:

(1) Symmetric return variability (up and down) of the portfolio around the return on a known benchmark, defined by

$$\text{Symmetric Risk} = \sum_{t=1}^T \omega_t [(\text{Portf Ret}_t - \text{Bch Ret}_t) - \text{Av}(\text{Portf Ret}_t - \text{Bch Ret}_t)]^2,$$

where  $\text{Portf Ret}_t$  = return on the chosen portfolio in period  $t$ ,  $\text{Bch Ret}_t$  = return on the benchmark in period  $t$ ,  $\text{Av}(\cdot)$  = average value of  $(\cdot)$  over periods  $t = 1, 2, \dots, T$ , and  $\omega_t$  = exponentially decaying weight assigned to period  $t$  (see below).

(2) Asymmetric downside risk (down-variability) with respect to a known benchmark, defined by

$$\text{Asymmetric Risk} = \sum_{t=1}^T \omega_t [(\text{Portf Ret}_t - \text{Bch Ret}_t)_- - \text{Av}(\text{Portf Ret}_t - \text{Bch Ret}_t)_-]^2,$$

where  $(\text{Portf Ret}_t - \text{Bch Ret}_t)_- = \min[0, (\text{Portf Ret}_t - \text{Bch Ret}_t)]$ , and  $\text{Av}(\text{Portf Ret}_t - \text{Bch Ret}_t)_- =$  average value of  $(\text{Portf Ret}_t - \text{Bch Ret}_t)_-$  over periods  $t = 1, 2, \dots, T$ .

(3) Asymmetric return variability around more than one benchmark. One can define a weighted average of several benchmarks.

(4) The classical Markowitz risk of portfolio variance, defined by

$$V = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j,$$

where  $\sigma = [\sigma_{ij}]$  is the covariance matrix between the returns of assets  $i$  and  $j$ .

In Opti-Money, we do not explicitly compute covariance-type matrices. For example, suppose we choose the portfolio variance  $V$  (risk measure 4) as the appropriate risk measure, i.e.,

$$V = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j,$$

where

$$\begin{aligned} \sigma_{ij} &= \frac{1}{T-1} \sum_{t=1}^T R_{t,i} R_{t,j} \\ &= \frac{1}{T-1} \sum_{t=1}^T \left( r_{t,i} - \frac{1}{T} \sum_{t=1}^T r_{t,i} \right) \left( r_{t,j} - \frac{1}{T} \sum_{t=1}^T r_{t,j} \right) \\ &\quad i, j = 1, 2, \dots, n, \end{aligned}$$

are the elements of the covariance matrix between the historical returns on assets  $i$  and  $j$ . Thus,

$$V = \frac{1}{T-1} \sum_{i=1}^n \sum_{j=1}^n \left( \sum_{t=1}^T R_{t,i} R_{t,j} \right) x_i x_j,$$

and this expression does not include the explicit covariance elements  $\sigma_{ij}$ .

### Using Exponentially Decaying Weights

Opti-Money estimates the risk measures of portfolios by observing asset returns in a fixed number  $T$  of historical periods (we usually set  $T = 36$  months). In the traditional method, the weight of each period is  $1/T$ . In our model, we have the option to use an exponential decay factor  $\lambda$  such that the latest observations carry the greatest weight in the risk estimate. The advantage of this method over the equally weighted model is that a shock (say, exceptionally high or low return) has an immediate effect on risk and the effect decays with time. In the equally weighted model, the effect of the shock would not change with time until its complete disappearance from the model. The exponentially decaying weight is given by

$$\omega_t = \lambda^{t-1} / \sum_{j=1}^T \lambda^{j-1}, \quad t = 1, 2, \dots, T.$$

For  $\lambda = 1$ , we obtain  $\omega_t = 1/T$ . An appropriate value of  $\lambda$  can be 0.95. Assume that  $t = 1$  was one month ago,  $t = 2$  was two months ago, etc. For  $T = 36$ , we get  $\omega_1 = 0.0594$ ,  $\omega_2 = 0.0564$ ,  $\dots$ ,  $\omega_{18} = 0.0248$ ,  $\dots$ ,  $\omega_{36} = 0.0099$ , compared to a fixed weight of  $1/36 = 0.0278$ . For a thorough discussion of decay factors, see Kim and Mina (2001).

### The Composition of the Optimal Portfolios

If the risk measure used in the model is relative to a benchmark and that benchmark is either explicitly represented in the asset-class universe of the model or can be well approximated by a few asset classes, then the Markowitz-type optimal portfolios are often unbalanced and exhibit corner solutions with zero fractions in many assets and unreasonably large fractions in others. Such solutions are unacceptable to portfolio managers. One remedy could be fixing or limiting the number of assets with nonzero fractions in the optimal portfolio, but this would convert the mathematical program into a mixed-integer nonlinear program (Bertsimas et al. 1999). Another approach to balance the portfolio could be artificially setting upper or lower limits or both to the asset fractions. The end result of this approach is that many assets would always sit at the predetermined limit. Both of these approaches were objectionable to us. Instead, we chose a more natural approach: We defined a value-weighted market portfolio of all asset classes participating in the Opti-Money model and modified our risk measures to consist of a sum of two weighted terms. The first term consists of one of the four risk measures defined above, and the second term is a Euclidean distance function in  $n$ -space between the portfolio chosen by the model and the market portfolio. Let  $m_j$ ,  $j = 1, 2, \dots, n$ , be the weights of the asset classes in the given market portfolio. Then, our modified risk measure is defined by (say, for symmetric return variability)

$$\begin{aligned} &\sum_{t=1}^T \omega_t [(\text{PortfRet}_t - \text{BchRet}_t) - \text{Av}(\text{PortfRet}_t - \text{BchRet}_t)]^2 \\ &+ \gamma \sum_{j=1}^n (x_j - m_j)^2, \end{aligned}$$

where  $\gamma$  is a small positive constant.

Thus, our objective function of risk minimization consists of both a variability-type risk minimization and a market-portfolio tracking term. By making an appropriate choice of the weight  $\gamma$  (usually very small), we obtained nicely balanced optimal portfolios without significant compromise in pure efficient-frontier optimality. By *efficient frontier*, we mean the usual definition, that is, the set of risk-expected-return combinations such that for any given value of risk, there is no higher feasible expected return, or for any given value of expected return, there is no lower feasible risk. The portfolios computed by the above-mentioned two-term minimizations constitute a balanced efficient frontier. The portfolios recommended to the customers by the Opti-Money model are drawn from these frontiers. We describe the construction of such a frontier next.

In summary, the Opti-Money model consists of various benchmark-related or classical Markowitz-type risk measures and a market portfolio, which we slightly track. In addition, we may constrain portfolio turnover, that is, given an existing portfolio of assets, the optimal portfolio obtained by the solution of Opti-Money can have no more than a certain percentage of new assets in it. For various classes of customers, constraints on the inclusion of certain assets should be imposed. For example, as we mentioned above, a customer may restrict the fraction of illiquid assets in the portfolio.

### Constructing an Efficient Frontier

During a routine monthly run of the Opti-Money system, thousands of balanced efficient frontiers are automatically constructed. Here is a description of constructing a balanced efficient frontier:

Given the  $n$  asset classes that can be included in the portfolio with their historical and expected future returns, the definition of the appropriate risk measure, the composition of the market portfolio, turnover, or customer-specific constraints,

(1) Find an optimal portfolio that will have minimum risk plus tracking error, irrespective of expected returns.

(2) Find an optimum portfolio that will have maximum expected return, irrespective of portfolio risk.

These two portfolios represent the two extreme points of the balanced efficient frontier in the risk-return plane. The portfolio obtained in Step 1 will have the lowest expected return and lowest risk; whereas the portfolio obtained in Step 2 will have the highest expected return and highest risk.

(3) Compute the expected returns of  $K$  intermediate points between the expected returns of the two portfolios obtained above. For example, space the  $K$  points at equal distances from each other.

(4) Solve  $K$  optimization problems of minimizing portfolio risk plus tracking error subject to expected portfolio returns obtained in Step 3. These  $K$  portfolios, along with the portfolios obtained in Steps 1 and 2, constitute a discrete approximation of a balanced efficient frontier. These portfolios, after a postoptimal adjustment, described next, are the portfolios that are recommended to the customers.

Because there are several customer-specific parameters, such as investment horizon, benchmark, liquidity level, tax status, and forecast modification, and for each parameter there are two to five discrete values, we have to construct a balanced efficient frontier for each possible combination of these parameter values. This is why we construct thousands of balanced efficient frontiers.

### Postoptimal Adjustment

Before submitting the optimal solution for actual execution of portfolio construction, we have to round the asset fractions to predetermined discrete values. Of course, this requirement could be incorporated in the overall nonlinear programming model, which would then become a mixed-integer program, but we decided to use continuous programming only. The method we use for optimal rounding is a simple computational procedure (Avriel 2003). A. C. Williams recently brought to our attention Balinski and Young's (1977) development of a similar rounding procedure in connection with proportional political representation.

The Opti-Money model is a constrained nonlinear program with continuous derivatives. For downside risk or for some choices of more than one benchmark, however, the model becomes a nonlinear program with discontinuous derivatives. For example, in case of downside risk, such expressions as  $\min(x, 0)$  appear in the objective function or the constraints.

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An abridged version of the closing remarks at the Edelman competition presentation by Mr. Abraham Harel, Chief Financial Officer of Bank Hapoalim: "The primary benefit, from the point of view of the customer, is the simple fact that the system creates a personal, tailor-made solution for each and every customer according to his or her objectives, forecasts, and attitude towards risk. The solution that is offered by Opti-Money means that the recommended portfolio is on the risk-return efficient frontier, or very close to it. In other words, the system is able to provide the best, or very close to best, solution in terms of risk-adjusted profitability.

"Turning now to the assessment of the contribution of Opti-Money to the bank's profitability, we examined the contribution to the bank's income of the private banking accounts of customers who obtained advice with the use of the system, as compared to customers who did not receive advice. Undoubtedly, the system contributes to the bank's income.

“There are additional, very considerable, indirect contributions as well, such as the establishment of mutual funds based on the Opti-Money system, and the development of additional decision-support systems for the CRMs.

“In conclusion, Opti-Money is an example of a win-win combination, where an analytical solution to an optimization problem enables us to create an advisory process for the benefit of the customers and the bank itself.”