

## Adding/Subtracting Demand and Supply

Always keep in mind that demand (or supply) curves are economically meaningful only for  $P \geq 0$  and  $Q \geq 0$ . This caution is trivial when we deal with graphs but is often neglected when we work with algebra.

### § Example 1

Given  $Q = a_0 - a_1P$ , we must remember that  $Q \geq 0$  and  $P \geq 0$ , that is,  $P \leq a_0/a_1$  and  $Q \leq a_0$ .

☛ Thus, when adding/subtracting demand (or supply) curves:

1. Add them horizontally, not vertically (assuming the good in question is a *private good*);
2. Delete the negative portions of price and quantity before, not after, adding.

### § Example 2

Suppose we want to add two demand curves

$$Q_1 = 10 - 5P \quad (P \geq 0, Q_1 \geq 0)$$

$$Q_2 = 6 - 2P \quad (P \geq 0, Q_2 \geq 0)$$

The result is a kinked curve characterized by:

$$Q = 0 \quad \text{when} \quad P > 3$$

$$Q = 6 - 2P \quad 2 < P < 3$$

$$Q = 16 - 7P \quad 0 < P < 2$$

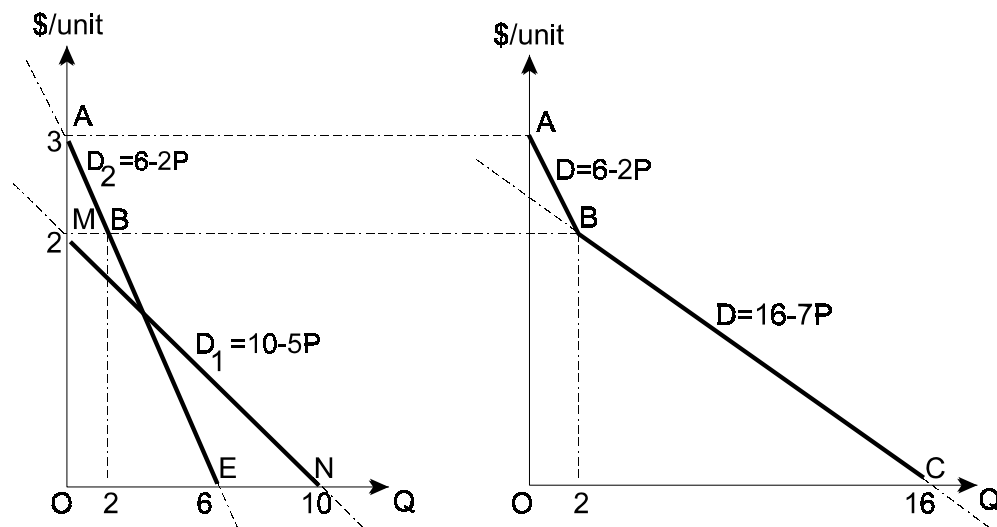


Figure 1.1. An Example of Demand Curve Summation

Note that when price is between \$2/unit and \$3/unit, the total demand consists only of the demand by group 2, thus the segment AB on the right panel is simply the duplicate of the segment AB on the left panel. Only when price falls below \$2/unit, which is group 1's reservation price, would total demand genuinely be the (horizontal) sum of  $D_1$  and  $D_2$  (segment BC on the right panel is the sum of segments MN and BE on the left panel). Observe that had we neglected to exclude the negative-quantity portion of  $D_1$ , the sum would extend beyond the straight line BC to include the dashed portion above point B in the right panel. This could cause grave errors in our analysis.

### § Example 3

In the market for commodity X there are two groups of buyers,  $\alpha$  and  $\beta$ . Their demands are given by  $Q_\alpha = 6 - 0.8P$  for group  $\alpha$ , and  $Q_\beta = 12 - 0.6P$  for group  $\beta$

- Determine the equilibrium market price and quantity if the supply curve is  $Q = 1.5P$
- Suppose the government now imposes a tax of 167% (to be precise,  $500/3$  per cent) of the pretax price of X. What would be the new equilibrium output and the price paid by the consumers?

• *Solution*

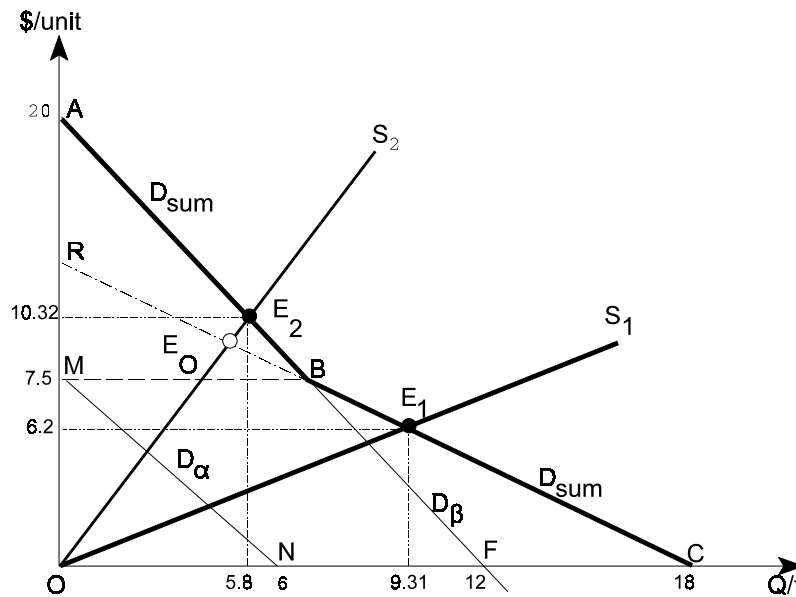


Figure 1.2. Another Example of Demand Curve Summation

First, we need to add the two demand curves to obtain market demand. This market demand ( $D_{sum}$ ) has two segments:

Segment AB:  $D_{sum} = 12 - 0.6P$  for  $7.5 < P < 20$

Segment BC:  $D_{sum} = 18 - 1.4P$  for  $0 < P < 7.5$

In the Figure,  $D_{sum}$  is the *kinked* curve ABC. Keep in mind that demand curves make economic sense only if price and quantity are non-negative; otherwise, the sum of the two demand curves would appear incorrectly to be the straight line RBC.

- In this case  $S_1$  is the supply curve. Equilibrium is determined by the intersection of the demand and

supply curves. From the Figure, clearly this intersection occurs (at point  $E_1$ ) in the segment BC of  $D_{sum}$ , hence

$$1.5P = 18 - 1.4P$$

which gives, at equilibrium:

$$P = 6.2$$

and  $Q = 9.31$  ■

b. The "plus tax" supply curve is  $S_2$ .<sup>1</sup> To obtain its equation, note that  $S_1$  can also be described by  $P = (1/1.5)Q$ . Now, the new price is the old price plus tax, hence  $P' = (5/3)P + P = (8/3)P = (8/4.5)Q$ . Thus  $S_2$  can be described by  $P' = (8/4.5)Q$ , or  $Q = (4.5/8)P'$ .

From the Figure, clearly the new equilibrium occurs (at point  $E_2$ ) in the AB segment of  $D_{sum}$ , at which only group  $\beta$  consumers are buying, hence

$$(4.5/8)P' = 12 - 0.6P'$$

which gives the consumer's price after tax:

$$P' = 10.32$$

and the new equilibrium quantity

$$Q' = 5.80$$
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Note that if we make the mistake of using RC as the entire  $D_{sum}$ , the new equilibrium would be erroneously located at  $E_0$  ( $P = 9.17$ ;  $Q = 5.15$ ). ■

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<sup>1</sup>It is misleading, however, to say that the supply curve "shifts" to the new position. There are actually two supply curves: the original one depicts what the sellers are willing to accept (as a result of his costs, technology, etc.) in the absence of tax, and the new one depicts what the sellers are willing to accept if he is also legally responsible to collect the tax in question for the government.