Reply to comment by Shlomo P. Neuman on "Spatial correlation of permeability in cross-stratified sediment with hierarchical architecture"

Robert W. Ritzi,[1] Zhenxue Dai,[1,2] David F. Dominic,[1] and Yoram N. Rubin[3]

Received 30 June 2005; revised 23 January 2006; accepted 6 February 2006; published 20 May 2006.

Citation: Ritzi, R. W., Z. Dai, D. F. Dominic, and Y. Rubin (2006), Reply to comment by Shlomo P. Neuman on ''Spatial correlation of permeability in cross-stratified sediment with hierarchical architecture,'' *Water Resour. Res.*, *42*, W05602, doi:10.1029/2005WR004402.

[1] We appreciate the commentary by *Neuman* [2006] and the opportunity for discussion and further clarification of our article [*Ritzi et al.*, 2004]. In his commentary, Neuman expounds upon his derivation of a model for the permeability semivariogram [*Neuman*, 2003, equation (53)] and describes aquifer architecture that would be represented by such a model. As requested by the Associate Editor, our reply focuses on illustrating the distinctions between the types of architecture represented by *Neuman* [2006] and *Ritzi et al.* [2004].

[2] *Ritzi et al.* [2004] considered the structure of permeability semivariograms arising in unconsolidated sedimentary deposits, the most common type of aquifer. Sedimentologists conventionally describe such deposits using a hierarchy of stratal unit types. Each of these stratal unit types is designated based on distinguishable characteristics, such as texture and geometry. A unit type defined at one hierarchical level is composed of smaller-scale unit types defined at the next lower level. Our approach makes a link between this hierarchical stratal architecture and the structure of the permeability semivariogram. To more clearly make this point we refer to Figure 1, which illustrates stratal architecture typically found in the sedimentological literature [e.g., *Bridge*, 2003]. Figure 1 shows a hierarchy of unit types designated within a sedimentary deposit formed as a braid bar within a river. The braid bar is composed of smaller-scale unit types such as large-scale inclined strata and cross-bar channel fills. In this example we refer to these as level II unit types within the hierarchy. Each of these level II unit types is, in turn, composed of smaller-scale level I unit types. For example, one large-scale inclined stratum (level II) is shown to be made up of level I units including small-scale cross strata composed of fine sand, planar strata composed of medium sand, and medium-scale cross strata composed of sandy gravel.

[3] An expanded hierarchy could be created by including unit types that exist at still larger or smaller scales, with any number of levels. For example, we could add a higher level in the hierarchy by representing larger-scale unit types such as point bars and major-channel fills, which occur with braid bars in channel belts. We could add a lower level in the hierarchy by representing smaller-scale stratal unit types, such as avalanche beds and interbeds, which occur within a set of cross strata [see *Ritzi et al.*, 2004]. *Dai et al.* [2005] showed that hierarchies may not be unique and that a unique hierarchal designation is not required for the approach presented by *Ritzi et al.* [2004].

[4] *Ritzi et al.* [2004] showed that the composite permeability semivariogram is related to a stratal hierarchy through their equation (1), in which the stratal architecture is represented using the conventional indicator geostatistics. Here we write a form of that equation for the two-level hierarchy illustrated in Figure 1. Consider two points $\underline{\boldsymbol{x}}$ and $\underline{\boldsymbol{x}}'$ which are separated by a lag vector $\underline{\boldsymbol{h}}$. Consider that location $\underline{\boldsymbol{x}}$ is within level I unit type k which, in turn, is within level II unit type i, and that $\underline{\boldsymbol{x}}'$ is within level I unit type m which is within level II unit type j. Thus, the tail of the lag vector at $\underline{\boldsymbol{x}}$ is in region type ik, and the head at $\underline{\boldsymbol{x}}'$ is in region type jm. The composite permeability semivariogram then can be written as:

$$\begin{aligned}\hat{\gamma}(\underline{\boldsymbol{h}}) = & \sum_i \sum_k \hat{\gamma}_{ik,ik}(\underline{\boldsymbol{h}})\hat{\boldsymbol{p}}_{ik}(\underline{\boldsymbol{h}})\hat{\boldsymbol{t}}_{ik,ik}(\underline{\boldsymbol{h}}) \\ & + \sum_i \sum_k \sum_{m\neq k} \hat{\gamma}_{ik,im}(\underline{\boldsymbol{h}})\hat{\boldsymbol{p}}_{ik}(\underline{\boldsymbol{h}})\hat{\boldsymbol{t}}_{ik,im}(\underline{\boldsymbol{h}}) \\ & + \sum_i \sum_{j\neq i} \sum_k \hat{\gamma}_{ik,jk}(\underline{\boldsymbol{h}})\hat{\boldsymbol{p}}_{ik}(\underline{\boldsymbol{h}})\hat{\boldsymbol{t}}_{ik,jk}(\underline{\boldsymbol{h}}) \\ & + \sum_i \sum_{j\neq i} \sum_k \sum_{m\neq k} \hat{\gamma}_{ik,jm}(\underline{\boldsymbol{h}})\hat{\boldsymbol{p}}_{ik}(\underline{\boldsymbol{h}})\hat{\boldsymbol{t}}_{ik,jm}(\underline{\boldsymbol{h}}) \end{aligned} \tag{1}$$

where the $\hat{\gamma}_{ik,jm}(\underline{\boldsymbol{h}})$ are the autosemivariogram and cross semivariograms [$ik = jm$ and $ik \neq jm$, respectively] for permeability as defined by level II and level I unit type. The $\hat{\boldsymbol{t}}_{ik,jm}(\underline{\boldsymbol{h}})$ are the transition probabilities, and the $\hat{\boldsymbol{p}}_{ik}(\underline{\boldsymbol{h}})$ are the proportions of region type ik in the tails of the lag pairs. The summations in this equation are grouped according to the type of transition. The first summation group on the RHS of equation (1) represents autotransitions at both levels II and I, and the other groups represent cross transitions: the second group represents cross transitions at level I, the third group at level II, and the fourth group at both levels.

[5] *Ritzi et al.* [2004] studied the relative contribution of each term toward defining the composite semivariogram, along the principal directions sampled, with data represent-

[1]Department of Geological Sciences, Wright State University, Dayton, Ohio, USA.
[2]Now at Earth and Environmental Sciences Division, Los Alamos National Laboratory, Los Alamos, New Mexico, USA.
[3]Department of Civil and Environmental Engineering, University of California, Berkeley, California, USA.

0043-1397/06/2005WR004402$09.00

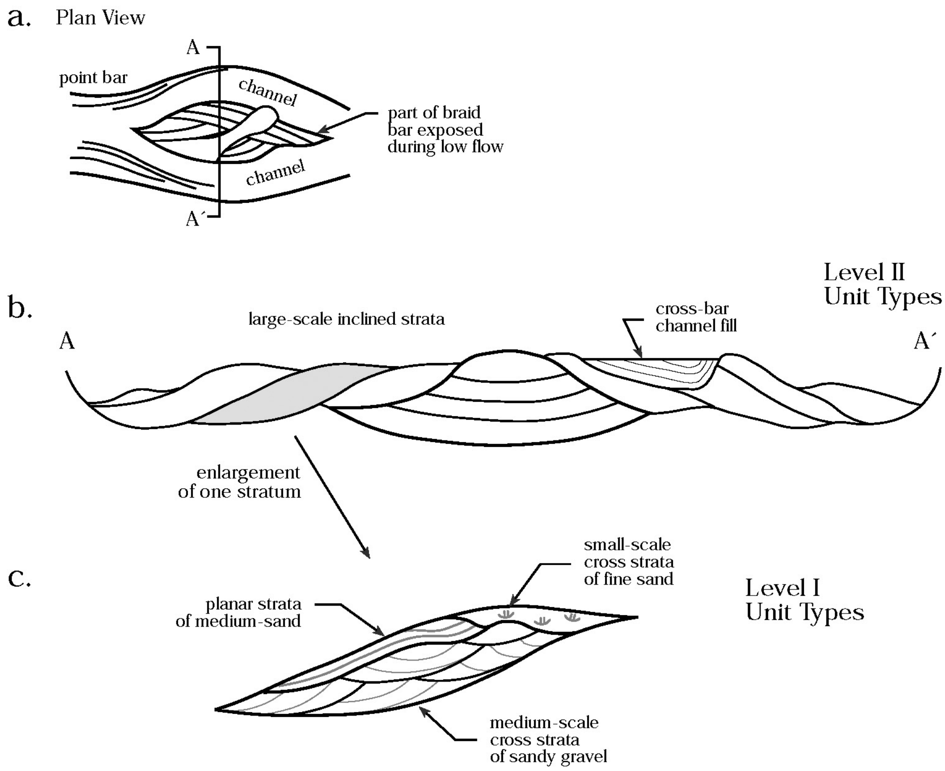

Figure 1. Stratal hierarchy for a braid bar. (a) Plan view of a channel belt showing a portion of a braid bar exposed during low flow. (b) Cross section illustrating level II unit types within the braid bar. (c) Detailed cross section of one large-scale inclined stratum showing that it comprises three level I unit types. Adapted from *Bridge* [2003, Figure 5.42].

ing a fluvial point-bar deposit. *Dai et al.* [2005] did so with data from the Skull Ridge member of the Tesuque Formation, an alluvial deposit in the Rio Grande Rift basin. In both cases, the shape and range of the composite semivariogram were mostly determined by the sum of the cross-transition terms (last three summation groups on the RHS of equation (1)), with negligible contribution from the autotransition terms (first summation group on the RHS of equation (1)). In both examples the semivariogram could be modeled by knowing just the cross-transition probabilities and the univariate statistics for permeability, further underscoring the importance of the cross-transition terms.

[6] *Neuman* [2003] presented a more specific form of our equation (1) as his equation (53). Here we show how the two are related. First we reduce the number of hierarchical levels below the composite scale to just one, with point $\underline{\boldsymbol{x}}$ in unit type i and point $\underline{\boldsymbol{x}}'$ in unit type j. We then change the lag vector from $\underline{\boldsymbol{h}}$ to an omnidirectional lag distance $\tilde{s}$ as defined by *Neuman* [2003], using an elliptical transformation of coordinates. This gives

$$\gamma(\tilde{s}) = \sum_i \gamma_{ii}(\tilde{s})\boldsymbol{p}_i(\tilde{s})\boldsymbol{t}_{ii}(\tilde{s}) + \sum_i \sum_{j\neq i} \gamma_{ij}(\tilde{s})\boldsymbol{p}_i(\tilde{s})\boldsymbol{t}_{ij}(\tilde{s}) \quad (2)$$

When comparing to *Neuman*'s [2003] equation (53):

$$\gamma(\tilde{s}) = \sum_i \boldsymbol{p}_i \gamma_{ii}(\tilde{s}) \quad (3)$$

it is clear that the cross-transition terms in the second summation group on the RHS of equation (2), which included the cross semivariograms $\gamma_{ij}(\tilde{s})\ \forall \boldsymbol{i} \neq \boldsymbol{j}$ and the cross-transition probabilities $\boldsymbol{t}_{ij}(\tilde{s})\ \forall \boldsymbol{i} \neq \boldsymbol{j}$, are dropped in equation (3), as pointed out by *Neuman* [2006]. Furthermore, given that the $\boldsymbol{t}_{ij}(\tilde{s})$ are unity at all lags in equation (3) and given that $\tilde{s}$ is defined as omnidirectional, the unit types must be of infinite length in all directions. Thus the equation (3) model represents an architecture in which there is one level for all unit types and all types exist at every point in space. This is a different architecture than the conventional stratal hierarchy depicted in Figure 1 and represented by our equation (1). Within any one of the hierarchical levels in Figure 1, the unit types have finite length and fill space as mutually exclusive regions. For example, if i and j represent level II unit types, a point in space cannot be within both a large-scale inclined stratum and a cross-bar channel fill. Equation (1) does not assume a particular measurement support scale nor does its use in the approach by *Ritzi et al.* [2004] and *Dai et al.* [2005] require one. However, defining the $\hat{\gamma}_{ik,jm}$ terms in equation (1) for our approach or the γ_{ii} terms in equation (3) for Neuman's approach requires, in both cases, a measurement support scale smaller than the smallest unit types that are defined by the architectural model.

[7] The issue from our article upon which *Neuman* [2006] has focused concerns the dropping of the cross-transition terms, as done by *Neuman* [2003] and as in other approaches in the literature [e.g. *Davis et al.*, 1997]. The

context was that when stratal hierarchies are delineated in sedimentary deposits based, in part, on differences in sediment texture (mean grain size and grain size sorting), the stratal unit types can give rise to a hierarchy of multiple permeability modes. If so, *Ritzi et al.* [2004] and *Dai et al.* [2005] have shown that in directions in which lags will cross stratal boundaries (which is true for all directions in Figure 1), the permeability semivariogram can be almost entirely defined by the sum of cross-transition terms (last three summation groups on the RHS of equation (1)). Indeed, in both examples presented in these articles, it was the autotransition terms rather than the cross-transition terms that could be assumed negligible when modeling the permeability semivariogram.

References

Bridge, J. S. (2003), *Rivers and Floodplains*, Blackwell, Malden, Mass.

Dai, Z., R. W. Ritzi Jr., and D. F. Dominic (2005), Improving permeability semivariograms with transition probability models of hierarchical sedimentary architecture derived from outcrop analog studies, *Water Resour. Res.*, *41*, W07032, doi:10.1029/2004WR003515.

Davis, J. M., J. L. Wilson, F. M. Phillips, and M. B. Gotkowitz (1997), Relationship between fluvial bounding surfaces and the permeability correlation structure, *Water Resour. Res.*, *33*(8), 1843–1854.

Neuman, S. P. (2003), Relationship between juxtaposed, overlapping, and fractal representations of multimodal spatial variability, *Water Resour. Res.*, *39*(8), 1205, doi:10.1029/2002WR001755.

Neuman, S. P. (2006), Comment on "Spatial correlation of permeability in cross-stratified sediment with hierarchical architecture" by Robert W. Ritzi, Zhenxue Dai, David F. Dominic, and Yoram N. Rubin, *Water Resour. Res.*, *42*, W05601, doi:10.1029/2005WR004256.

Ritzi, R. W., Z. Dai, D. F. Dominic, and Y. Rubin (2004), Spatial correlation of permeability in cross-stratified sediment with hierarchical architecture, *Water Resour. Res.*, *40*, W03513, doi:10.1029/2003WR002420.

Z. Dai, Earth and Environmental Sciences Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA.

D. F. Dominic and R. W. Ritzi, Department of Geological Sciences, Wright State University, 3640 Colonel Glenn Highway, Dayton, OH 45435, USA. (rritzi@wright.edu)

Y. N. Rubin, Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720-1710, USA.