PROBLEM 1 (12 pts)
A cylindrical resistance wire (radius $R$, thermal conductivity $k$) is used to boil water. Heat is generated in the wire as a result of resistance heating at a volumetric rate $\dot{e}_g = a/r$, where $a$ is a constant. The heat generated is transferred to water at temperature $T_\infty$ by convection with an average heat transfer coefficient $h$.

The heat diffusion equation in cylindrical coordinates is given below:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( k \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{e}_g$$

a) (3 pts) Assuming: (i) steady-state conditions, (ii) one-dimensional heat transfer, eliminate the appropriate term(s) on the equation above by indicating the assumption used. Write the reduced equation for heat conduction in the wire below.

b) (6 pts) By solving the reduced equation and using appropriate boundary conditions, derive an expression for the temperature distribution within the wire $T(r, R, k, T_\infty, h)$

hint: the temperature at the center of the wire cannot be infinite
c) (3 pts) Derive an expression for the total heat rate dissipated in the water by a wire of length $L$. 
**PROBLEM 2 (8 pts)**

A rectangular central processing unit (CPU, length \( L = 25 \text{ mm} \), height \( H = 10 \text{ mm} \)) operates steadily at a maximum temperature \( T_b \). Heat from the CPU is dissipated from the CPU surface via convection with the ambient air (temperature \( T_{\infty} = 20^\circ \text{C} \), convection coefficient \( h = 40 \text{ W/m}^2\text{K} \)) and via 4 cylindrical fins (diameter \( D = 1.5 \text{ mm} \), thermal conductivity \( k = 17 \text{ W/m-K} \)). The left surface of the CPU is insulated and the heat sink assembly is exposed to ambient air.

**a)** (3 pts) Complete the equivalent thermal circuit below for the CPU/fins assembly, assuming one-dimensional heat transfer. Include the following labels: \( \dot{Q}_f, \dot{Q}_{\text{conv}}, R_f, R_{\text{conv}} \), where \( \dot{Q}_f \) is the heat transfer rate on one fin, \( \dot{Q}_{\text{conv}} \) is the heat transfer rate by convection from the CPU surface to the air, and \( R_f, R_{\text{conv}} \) are the thermal resistances for heat transfers through one fin and between the CPU and the air, respectively.

**b)** (1 pt) Using the circuit from part a), derive a relationship between \( \dot{Q}, \dot{Q}_f, \dot{Q}_{\text{conv}} \).
c) (4 pts) Using the relationship from part b), assuming very long fins and using the table below, calculate the maximum CPU heat transfer rate $\dot{Q}$ to limit its operating temperature to the maximum allowable temperature $T_b = 75^\circ C$.

<table>
<thead>
<tr>
<th>Case</th>
<th>Tip Condition $(x = L)$</th>
<th>Temperature Distribution $\theta/\theta_s$</th>
<th>Fin Heat Transfer Rate $q$</th>
</tr>
</thead>
</table>
| A    | Convection heat transfer:  
$\dot{h}(L) = -k \frac{d\theta}{dx}|_{x=L}$  
$cosh m(L - x) + (h/mk) sinh m(L - x)$  
$cosh mL + (h/mk) sinh mL$ | $\frac{M sinh mL + (h/mk) cosh mL}{cosh mL + (h/mk) sinh mL}$ | (3.75) |
| B    | Adiabatic:  
$\theta(L) = \theta_s$  
$cosh m(L - x)$  
$cosh mL$ | $\frac{M tanh mL}{cosh mL}$ | (3.80) |
| C    | Prescribed temperature:  
$\theta(L) = \theta_s$  
$(\theta_s/\theta_s) sinh m x + sinh m(L - x)$  
$sinh mL$ | $\frac{M (cosh mL - \theta_s/\theta_s)}{sinh mL}$ | (3.83) |
| D    | Infinite fin $(L \rightarrow \infty)$:  
$\theta(L) = 0$  
$e^{-mx}$ | $M$ | (3.85) |

$\theta = T - T_m$  
$m^2 = hP/kA_c$  
$\theta_b = \theta(0) = T_b - T_m$  
$M = \sqrt{hP/A_c \theta_b}$