Problem 1

A cylindrical shaft (radius $R_1$) rotates at angular velocity $\omega$ in a bearing (radius $R_2$) filled with oil (thermal conductivity $k$, viscosity $\mu$, specific heat $c$). At steady operating conditions, the bearing is cooled externally and maintained at a temperature $T_0$. The surface of the shaft is insulated. It can be shown that the velocity field in the fluid is expressed as:

$$v_\theta(r) = \frac{R_1^2 \omega}{R_1^2 - R_2^2} \left( r - \frac{R_2^2}{r} \right).$$

The energy equation in cylindrical coordinates is given below:

$$\rho c_p \left( \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_\theta \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + \ldots$$

$$\ldots + \mu \left\{ 2 \left( \frac{\partial v_r}{\partial r} \right)^2 + \frac{1}{r^2} \left( v_r + \frac{\partial v_\theta}{\partial \theta} \right)^2 + \left( \frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_z}{\partial z} \right)^2 \right\} + q$$

1- Simplify the energy equation under the following assumptions: (i) steady flow, (ii) 2D flow, (iii) axisymmetric flow, (iv) no heat generation, and (v) $v_r = v_z = 0$. Eliminate the appropriate terms in the equations by specifying the assumption that was used.

2- Using the reduced form of the energy equation, derive an expression for the temperature distribution $T$ in the oil gap.

3- Derive an expression for the maximum oil temperature.

4- Derive an expression for the rate of heat transfer per unit length to the bearing.

Problem 2

Consider airflow over a flat plate of length $L = 1$ m under conditions for which transition occurs at $x_c = 0.5$ m based on the critical Reynolds number, $Re_{x,c} = 5 \times 10^5$. 
In the laminar and turbulent regions, the local convection coefficients are \( h_{\text{lam}}(x) = C_{\text{lam}} x^{-0.2} \) and \( h_{\text{turb}}(x) = C_{\text{turb}} x^{-0.2} \), respectively, where, at \( T = 350 \) K, \( C_{\text{lam}} = 8.845 \) W/m\(^{3/2}\)K, \( C_{\text{turb}} = 49.75 \) W/m\(^{3/2}\)K, and \( x \) has units of m.

1. Evaluating the thermophysical properties of air at 350 K, determine the air velocity.

2. Develop an expression for the average convection coefficient, \( \overline{h}_{\text{lam}}(x) \), as a function of distance from the leading edge, \( x \), for the laminar region \( 0 \leq x \leq x_c \).
   
   Hint: the average value \( \overline{f} \) of any quantity \( f \) is calculated as: \( \overline{f} = \frac{1}{x} \int_0^x f \, dx \)

3. Develop an expression for the average convection coefficient, \( \overline{h}_{\text{turb}}(x) \), as a function of distance from the leading edge, \( x \), for the turbulent region \( x_c \leq x \leq L \).

4. Using Matlab or Excel, plot on the same graph the local and average convection coefficients, \( h_x \) and \( \overline{h}_x \), respectively, as a function of \( x \) for \( 0 \leq x \leq L \).

**Problem 3**

An airfoil with a characteristic length \( L_1 = 0.2 \) ft is placed in airflow at \( p = 1 \) atm and \( T_\infty = 60^\circ\text{F} \) with free stream velocity \( V_1 = 150 \) ft/s and convection heat transfer coefficient \( h_1 = 21 \) Btu/h·ft\(^2\)·°F.

A second larger airfoil with a characteristic length \( L_2 = 0.4 \) ft is placed in the airflow at the same air pressure and temperature, with free stream velocity \( V_2 = 75 \) ft/s. Both airfoils are maintained at a constant surface temperature \( T_s = 180^\circ\text{F} \).

Determine the heat flux from the second airfoil.

[solution: \( q = 1260 \) Btu/h·ft\(^2\)]