Problem 1

The steady-state temperature distribution in a semi-transparent material (thermal conductivity: \( k \); thickness: \( L \)) exposed to laser irradiation is of the form:

\[
T(x) = -\frac{A}{ka^2} e^{-ax} + Bx + C,
\]

where \( A, a, B, C \) are known constants.

For this situation, radiation absorption in the material is manifested by a distributed heat generation term, \( \dot{\varepsilon}_g(x) \).

1- Obtain expressions for the conduction heat fluxes at the front and rear surfaces

2- Derive an expression for \( \dot{\varepsilon}_g(x) \)

3- Derive an expression for the rate \( \dot{g}_{abs} \) at which radiation is absorbed in the entire material, per unit surface area. Express your result in terms of the known constants for the temperature distribution, the thermal conductivity of the material, and its thickness.

\[
[\text{solution: } \dot{g}_{abs} = \frac{A}{a} \left( 1 - e^{-axL} \right) ]
\]

Problem 2

A firefighter’s protective clothing, referred to as a turnout coat, is typically constructed as an ensemble of 3 layers separated by air gaps:

Representative dimensions and thermal conductivities for the layers are as follows:

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (mm)</th>
<th>( k ) (W/m·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air gap</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Shell (s)</td>
<td>0.8</td>
<td>0.047</td>
</tr>
<tr>
<td>Moisture barrier (mb)</td>
<td>0.55</td>
<td>0.012</td>
</tr>
<tr>
<td>Thermal liner (tl)</td>
<td>3.5</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Heat is transferred in the air gaps by conduction and radiation exchange through the stagnant air. The linearized radiation coefficient for a gap may be approximated as:
\[ h_{\text{rad}} = \sigma(T_1 + T_2)(T_1^2 + T_2^2) \approx 4\sigma T_{\text{avg}}^3, \]

where \( T_{\text{avg}} \) represents the average temperature of the surfaces comprising the gap, and the radiation flux across the gap may be expressed as:

\[ \dot{q}_{\text{rad}} = h_{\text{rad}}(T_1 - T_2) \]

1. Represent the turnout coat by a thermal circuit, labeling all the thermal resistances. Calculate and tabulate the thermal resistances per unit area (m\(^2\cdot\text{K}/\text{W}\)) for each of the layers, as well as for the conduction and radiation processes in the gaps. Assume that a value of \( T_{\text{avg}} = 470 \text{ K} \) may be used to approximate the radiation resistance of both air gaps.

2. Comment on the relative magnitudes of the resistances.

3. In a typical fire environment, the radiant heat flux on the fire-side of the turnout coat is \( \dot{q}_{\text{rad}} = 0.25 \text{ W/cm}^2 \). What is the outer surface temperature of the turnout coat if the inner surface temperature is 66°C, a condition that would result in burn injury?  

[solution: \( T_o = 534^\circ\text{C} \)]