Problem 1
1- Mass flow rate

The mass flow rate is calculated as:
\[ \dot{m} = \rho A \bar{V}, \]
where \( \rho \) is the fluid density, \( A \) is the pipe cross-sectional area, and \( \bar{V} \) is the average fluid velocity.

\[ \bar{V} = \frac{1}{A} \int_A \dot{V} \, dA = \frac{4}{\pi D^2} \int_{r=0}^{D/2} 0.2 \left[ 1 - \left( \frac{2r}{D} \right)^2 \right] 2\pi r \, dr \]

\[ \Rightarrow \bar{V} = \frac{4}{\pi D^2} 0.4\pi \int_{r=0}^{D/2} \left( r - \frac{4r^3}{D^2} \right) \, dr \]

\[ \Rightarrow \bar{V} = \frac{4}{\pi D^2} 0.4\pi \left[ \frac{r^2}{2} - \frac{r^4}{D^2} \right]_{0}^{D/2} \]

\[ \Rightarrow \bar{V} = \frac{1.6}{16} = 0.1 \text{ m/s} \]

Therefore:

\[ \dot{m} = \frac{0.1\pi \rho D^2}{4} \Rightarrow \dot{m} = 6.052 \times 10^{-3} \text{ kg/s} \]

2- Surface heat flux

The surface heat flux can be determined using Newton’s law of cooling:
\[ \dot{q}_s = h(T_s - T_m) \]

The surface temperature is: \( T_s = T(r = \frac{D}{2}) = 250 + 200 = 450 \text{ K} \)

The mean fluid temperature is calculated as:

\[ T_m = \frac{2}{VR^2} \int_{r=0}^{D/2} T(r) \dot{V} \, rdr \]

\[ \Rightarrow T_m = \frac{8}{VD^2} \int_{r=0}^{D/2} \left[ 250 + 200 \left( \frac{2r}{D} \right)^3 \right] \left[ 0.2 \left[ 1 - \left( \frac{2r}{D} \right)^2 \right] \right] r \, dr \]
\[ T_m = \frac{16}{D^2}(18.48D^2) \Rightarrow T_m = 295.7\, \text{K} \]

Substituting back in Newton’s law of cooling:

\[ q_s = 100(450 - 295.7) \Rightarrow q_s = 15.4\, \text{kW/m}^2 \]

**Problem 2**

1. **Average heat transfer coefficient**

We first determine the properties of the working fluid at an appropriate average temperature.

\[ \bar{T} = \frac{T_i + T_o}{2} = \frac{10 + 24}{2} = 17^\circ\text{C} \]

Using the properties of saturated water:

<table>
<thead>
<tr>
<th>Temp. ℃</th>
<th>Saturation Pressure (kPa)</th>
<th>Density (ρs) kg/m³</th>
<th>Enthalpy of Vaporization (Ci) J/kg</th>
<th>Specific Heat (cp) J/kg K</th>
<th>Thermal Conductivity (k) W/m-K</th>
<th>Dynamic Viscosity (µs) kg/m s</th>
<th>Prandtl Number</th>
<th>Volume Expansion Coefficient (β) 1/K</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.6113</td>
<td>999.8</td>
<td>0.0048</td>
<td>2501</td>
<td>4217</td>
<td>1854</td>
<td>0.561</td>
<td>0.0171</td>
</tr>
<tr>
<td>5</td>
<td>0.8721</td>
<td>999.9</td>
<td>0.0068</td>
<td>2490</td>
<td>4205</td>
<td>1857</td>
<td>0.571</td>
<td>0.0173</td>
</tr>
<tr>
<td>10</td>
<td>1.2376</td>
<td>999.7</td>
<td>0.0094</td>
<td>2478</td>
<td>4194</td>
<td>1862</td>
<td>0.580</td>
<td>0.0176</td>
</tr>
<tr>
<td>15</td>
<td>1.7051</td>
<td>999.1</td>
<td>0.0128</td>
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<td>4185</td>
<td>1863</td>
<td>0.589</td>
<td>0.0179</td>
</tr>
<tr>
<td>20</td>
<td>2.339</td>
<td>998.0</td>
<td>0.0173</td>
<td>2454</td>
<td>4182</td>
<td>1867</td>
<td>0.598</td>
<td>0.0182</td>
</tr>
<tr>
<td>25</td>
<td>3.169</td>
<td>997.0</td>
<td>0.0231</td>
<td>2442</td>
<td>4180</td>
<td>1870</td>
<td>0.607</td>
<td>0.0186</td>
</tr>
<tr>
<td>30</td>
<td>4.246</td>
<td>996.0</td>
<td>0.0304</td>
<td>2431</td>
<td>4178</td>
<td>1875</td>
<td>0.615</td>
<td>0.0189</td>
</tr>
</tbody>
</table>

Using linear interpolation between \( T = 15^\circ\text{C} \) and \( T = 20^\circ\text{C} \):

\( \rho = \rho_{15^\circ} + \frac{T - 15}{20 - 15}(\rho_{20^\circ} - \rho_{15^\circ}) \Rightarrow \rho = 999.1 + \frac{17 - 15}{20 - 15}(998 - 999.1) \Rightarrow \rho = 998.7 \, \text{kg/m}^3 \)

\( c_p = c_{p15^\circ} + \frac{T - 15}{20 - 15}(c_{p20^\circ} - c_{p15^\circ}) \Rightarrow c_p = 4185 + \frac{17 - 15}{20 - 15}(4182 - 4185) \Rightarrow c_p = 4183.8 \, \text{J/kg} \cdot \text{K} \)

The convection coefficient is obtained from Newton’s law of cooling. For the case of a constant surface temperature, it is expressed as:

\[ \bar{h}_L \cdot A \frac{\Delta T_o}{\Delta T} = \bar{h}_L \Rightarrow \bar{h}_L = \frac{\dot{Q}}{A \Delta T} \Rightarrow \bar{h}_L = \frac{\dot{Q}}{\pi DL \Delta T} \]

The total rate of heat transfer across the entire tube is calculated by considering an energy balance:

\[ \dot{Q} = \dot{m}c_p(T_o - T_i) \]

where the mass flow rate is expressed as:
\[ \dot{m} = \rho A \bar{V} = \rho \left( \pi \frac{D^2}{4} \right) \bar{V} \]

Substituting these expressions in the convection coefficient formulation:

\[ \bar{h}_L = \rho \left( \pi \frac{D^2}{4} \right) \bar{V} \frac{c_p(T_o - T_i)}{\pi DL} \frac{\ln \frac{\Delta T_o}{\Delta T_i}}{\Delta T_o - \Delta T_i} \]

\[ \Rightarrow \bar{h}_L = \rho D\bar{V} \frac{c_p(T_o - T_i)}{4L} \frac{\ln \frac{T_o - T_i}{T_i}}{T_o - T_i} \]

**Numerical application:**

\[ \bar{h}_L = 998.7 \times 0.012 \times \frac{4183.8(24 - 10)}{4 \times 5} \frac{\ln \frac{30 - 24}{10 - 24}}{30 - 10} \]

\[ \Rightarrow \bar{h}_L = 12.1 \text{ kW/m}^2 \cdot \text{K} \]

2- Number of tubes

The rate of energy to condense steam is calculated as:

\[ \dot{Q}_{\text{cond}} = \dot{m}_{\text{cond}} h_{fg} \]

This should be equal to the total rate of heat transfer generated by \( N \) tubes:

\[ N\dot{Q} = Ni c_p (T_o - T_i) = \dot{m}_{\text{cond}} h_{fg} \]

\[ \Rightarrow N\rho \left( \pi \frac{D^2}{4} \right) \bar{V} c_p (T_o - T_i) = \dot{m}_{\text{cond}} h_{fg} \]

\[ \Rightarrow N = \frac{4\dot{m}_{\text{cond}} h_{fg}}{\rho \pi D^2 \bar{V} c_p (T_o - T_i)} \]

**Numerical application:**

\[ N = \frac{4 \times 0.15 \times 2431}{998.7 \times \pi \times (0.012)^2 \times 4 \times 4183.8(24 - 10)} \]

\[ \Rightarrow N = 13.8 \]
3- Number of tubes vs. mean water velocity

![Graph](image)

**Problem 3**

1- **Mean temperature**

To determine the mean temperature, we consider an energy balance on a small control volume of the pipe:

\[
\dot{m}c_p \left[ (T_m + \frac{dT_m}{dz} \, dz) - T_i \right] = \dot{Q}_s \, \pi D \, dz
\]

\[
\Rightarrow \dot{m}c_p \frac{dT_m}{dz} = \dot{Q}_s \pi D
\]

\[
\Rightarrow dT_m = \frac{\dot{Q}_s \pi D}{\dot{m}c_p} \, dz
\]
Integrating from $z = 0$ to $z$:

$$T_m(z) - T_{m,i} = \frac{\pi D}{m c_p} \int_0^z a_z \, dz$$

$$\Rightarrow T_m(z) = T_i + \frac{\pi Da}{2mc_p} z^2$$

2- Outlet mean temperature
Using the expression obtained in part 1:

$$T_{m,o} = T_m(L) = T_i + \frac{\pi Da}{2mc_p} L^3$$

Numerical application:

$$T_{m,o} = 25 + \frac{\pi \times 0.025 \times 400}{2 \times 0.1 \times 4178} \Rightarrow T_{m,o} = 44.9^\circ C$$

3- Uniform surface heat flux
In the presence of a uniform surface heat flux $\dot{q}_s$, the overall energy balance is expressed as:

$$m c_p (T_o - T_i) = \dot{q}_s \pi DL$$

$$\Rightarrow \dot{q}_s = \frac{mc_p (T_o - T_i)}{\pi DL}$$

Numerical application:

$$\dot{q}_s = \frac{0.1 \times 4178 \times (44.9 - 25)}{\pi \times 0.025 \times 23} \Rightarrow \dot{q}_s = 4600 \text{ W/m}^2$$