Problem 1

1- The no-slip condition describes the velocity of fluid particles along surfaces.

Since the inner and outer cylinders are stationary, fluid particles located along their surface are not moving. Therefore, the no-slip condition along these surfaces is formulated mathematically as:

\[
\begin{align*}
    v_z(r = R_o) &= 0 \\
    v_z(r = R_i) &= 0
\end{align*}
\]

We need to verify that these equalities are satisfied by the velocity profile provided in the problem.

Inner cylinder:

\[
v_z(R_o) = \frac{1}{4\mu} \left( \frac{\Delta p}{L} \right) \left[ \frac{R_o^2 - R_i^2}{\ln \left( \frac{R_i}{R_o} \right)} \right] - \frac{R_o^2 - R_i^2}{\ln \left( \frac{R_i}{R_o} \right)} \ln \left( \frac{R}{R_o} \right) = 0
\]

Outer cylinder:

\[
v_z(R_o) = \frac{1}{4\mu} \left( \frac{\Delta p}{L} \right) \left[ R_i^2 - R_o^2 - \left( R_o^2 - R_i^2 \right) \ln \left( \frac{R_i}{R_o} \right) \right] - \frac{R_o^2 - R_i^2}{\ln \left( \frac{R_i}{R_o} \right)} \ln \left( \frac{R}{R_o} \right) = 0
\]

\[
\Rightarrow v_z(R_o) = \frac{1}{4\mu} \left( \frac{\Delta p}{L} \right) \left[ R_i^2 - R_o^2 + \left( R_o^2 - R_i^2 \right) \right] = 0
\]

Conclusion: \( \{v_z(r = R_o) = 0\} \) The no-slip condition is satisfied.

2- The shear stress at any point in the fluid gap is expressed as: \( \tau = \mu \frac{\partial v_z}{\partial r} \)

Using the velocity profile provided in the problem:

\[
\tau(r) = \mu \frac{\partial}{\partial r} \left[ \frac{1}{4\mu} \left( \frac{\Delta p}{L} \right) \left[ R_i^2 - r^2 - \frac{R_o^2 - R_i^2}{\ln \left( \frac{R_i}{R_o} \right)} \ln \left( \frac{r}{R_i} \right) \right] \right] = \frac{\Delta p}{4L} \frac{\partial}{\partial r} \left[ R_i^2 - r^2 - \frac{R_o^2 - R_i^2}{\ln \left( \frac{R_i}{R_o} \right)} \ln \left( \frac{r}{R_i} \right) \right]
\]
\[
\tau(r) = \frac{\Delta p}{4L} \left( -2r - \frac{R_o^2 - R_i^2}{R_i \ln \left( \frac{R_i}{R_o} \right) r} \right) = -\frac{\Delta p}{4L} \left( 2r + \frac{R_o^2 - R_i^2}{r \ln \left( \frac{R_i}{R_o} \right)} \right)
\]

The radial location \( r \) where the shear stress is zero (minimum value) can be found by solving:

\[
2r + \frac{R_o^2 - R_i^2}{r \ln \left( \frac{R_i}{R_o} \right)} = 0 \Rightarrow 2r^2 + \frac{R_o^2 - R_i^2}{\ln \left( \frac{R_i}{R_o} \right)} = 0 \Rightarrow 2r^2 = \frac{R_o^2 - R_i^2}{\ln \left( \frac{R_i}{R_o} \right)}
\]

Therefore:

\[
r = \left[ \frac{R_o^2 - R_i^2}{2 \ln \left( \frac{R_i}{R_o} \right)} \right]^{1/2}
\]

**Numerical application:** \( r = 13.7 \) mm

3. The viscous force exerted on a surface of area \( A \) is: \( F = \tau A \)

**Viscous force along the inner cylinder:**

\[
F_i = \tau(r = R_i) A_i,
\]

where \( A_i \) is the surface area of contact between the inner cylinder and the fluid (i.e., lateral surface of the inner cylinder).

Therefore:

\[
F_i = -\frac{\Delta p}{4L} \left( 2R_i + \frac{R_o^2 - R_i^2}{R_i \ln \left( \frac{R_i}{R_o} \right)} \right) (2\pi R_i L)
\]

\[
\Rightarrow F_i = -\frac{\pi R_i \Delta p}{2} \left( 2R_i + \frac{R_o^2 - R_i^2}{R_i \ln \left( \frac{R_i}{R_o} \right)} \right)
\]

**Numerical application:** \( F_i = 63.4 \) N (in magnitude)
Viscous force along the outer cylinder:

\[ F_o = \tau (r = R_o) A_o, \]

where \( A_o \) is the surface area of contact between the outer cylinder and the fluid (i.e., lateral surface of the outer cylinder).

Therefore:

\[ F_o = -\frac{\Delta p}{4L} \left( 2R_o + \frac{R_o^2 - R_i^2}{R_o \ln \left( \frac{R_i}{R_o} \right)} \right) (2\pi R_o L) \]

\[ \Rightarrow F_o = -\frac{\pi R_o \Delta p}{2} \left( 2R_o + \frac{R_o^2 - R_i^2}{R_o \ln \left( \frac{R_i}{R_o} \right)} \right) \]

**Numerical application:** \( F_o = 172 \text{ N (in magnitude)} \)

\[ \Delta p \pi (R_o^2 - R_i^2) = (p_{in} - p_{out}) \left( \pi R_o^2 - \pi R_i^2 \right) = p_{in} \left( \pi R_o^2 - \pi R_i^2 \right) - p_{out} \left( \pi R_o^2 - \pi R_i^2 \right) \]

Therefore, the term \( \Delta p \pi (R_o^2 - R_i^2) \) represents the net pressure force acting on the fluid gap.

Along the axial direction, the fluid contained within the annular gap experiences this pressure force as well as the viscous force exerted by the inner cylinder and the viscous force exerted by the outer cylinder. The viscous forces exerted by the cylinders oppose the motion of the fluid and are therefore directed from right to left.

\[ F_{in} - F_i - F_{out} - F_o = 0 \implies F_{in} - F_{out} - (F_i + F_o) = 0 \]

If the fluid does not accelerate \( (v_z \neq v_z(z)) \), the viscous force should balance the pressure force.

Balance of forces: \( F_{in} - F_i - F_{out} - F_o = 0 \implies F_{in} - F_{out} - (F_i + F_o) = 0 \)
Therefore, we expect: 
\[ \Delta p \pi (R_o^2 - R_i^2) = F_o + F_i \]

**Numerical application:**

\[
\begin{align*}
\Delta p \pi (R_o^2 - R_i^2) &= 236 \text{N} \\
F_o + F_i &= 172 + 63.4 = 235.4 \text{N}
\end{align*}
\]

Therefore, we have just demonstrated that the net pressure force balances the net viscous force.

### Problem 2

1. The condition of static equilibrium for the needle results from a balance between the weight of the needle and the surface tension force exerted by the fluid along its side:

\[ W + T = 0 \]

In terms of magnitudes:

\[ W = T \]

The weight of the needle is:

\[ W = \rho_s \left( \frac{\pi D^2}{4} L \right) g \]

The surface tension force is exerted along the linear interface between the fluid, the air and the cylindrical needle:

\[ T = 2\sigma L \]

Therefore:

\[ \rho_s \left( \frac{\pi D^2}{4} L \right) g = 2\sigma L \]
2- The needle will float if the surface tension force is larger than the weight of the needle:

\[ 2\sigma L \geq \rho_s \left( \frac{\pi D^2}{4} - L \right) g \]

\[ \Rightarrow D \leq \sqrt{\frac{8\sigma}{\pi \rho_s g}} = \sqrt{\frac{8\sigma}{\pi \rho_g g}} \]

The mass of the needle is expressed in terms of the specific gravity:

\[ SG = \frac{\rho_s}{\rho_{water}} \Rightarrow \rho_s = SG \rho_{water} \]

Substituting in the equation above:

\[ D \leq \sqrt{\frac{8\sigma}{\pi SG \rho_{water} g}} \]

**Numerical application:** \( D \leq 1.55 \text{ mm} \)

Only the 1-mm needles will float. The length of the needle does not impact this result.

### Problem 3

1- The Mach number is defined as: \( Ma = \frac{V}{c} \)

Assuming that air behaves as an ideal gas, the speed of sound can be expressed as:

\[ c = \sqrt{krT} \]

where \( k \) is the specific heat ratio, \( R \) is the gas constant (\( r = R/M \), with \( R = 8314 \text{ J/kgmol-K} \) and \( M \) is the gas molecular mass) and \( T \) is the gas absolute temperature (\( T = 50^\circ \text{F} = 283.15 \text{K} \)).

Using Table A.6 in the textbook:

\[ k = 1.40 \quad M = 28.98 \text{ kg/mol} \]

Therefore:

\[ r = R/M = 8314/28.98 = 286.9 \text{ J/kg-K} \]

\[ \Rightarrow c = 337.2 \text{ m/s} = 1106.4 \text{ ft/s} \]

**Numerical application:** \[ Ma = \frac{1173.3}{1106.4} = 1.06 \] (supersonic flow)

2- Since \( Ma > 0.3 \), compressibility effects cannot be neglected.