SUPERPOSITION OF BASIC PLANE POTENTIAL FLOWS

**Doublet**
A doublet is the combination of a source and sink

- **Combined velocity potential and streamfunction**

<table>
<thead>
<tr>
<th>Streamfunction</th>
<th>Sink</th>
<th>Source</th>
</tr>
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<tbody>
<tr>
<td>$\psi = \frac{-m}{2\pi} \theta$</td>
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Therefore, the combination of a uniform flow and a doublet is expressed in cylindrical coordinates as:

Streamfunction: $\psi = \frac{-m}{2\pi} (\theta_1 - \theta_2)$

Velocity potential: $\varnothing = \frac{-m}{2\pi} (\ln r_1 - \ln r_2)$

From the combined streamfunction:

$$\tan (\theta_1 - \theta_2) = \tan \left( \frac{-2\pi \psi}{m} \right)$$

Using the identity $\tan (\theta_1 - \theta_2) = \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$, the expression can be rewritten:

$$\frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2} = \tan \left( \frac{-2\pi \psi}{m} \right)$$
where:
\[
\begin{align*}
\tan \theta_1 &= \frac{r \sin \theta}{r \cos \theta - a} \\
\tan \theta_2 &= \frac{r \sin \theta}{r \cos \theta + a}
\end{align*}
\]

Therefore:
\[
\tan (\theta_1 - \theta_2) = \frac{2a r \sin \theta}{r^2 - a^2} = \tan \left(- \frac{2\pi \psi}{m} \right)
\]

The streamfunction is obtained by taking the inverse tangent:
\[
\psi = -\frac{m}{2\pi} \arctan \left( \frac{2a r \sin \theta}{r^2 - a^2} \right)
\]

The expression above is the streamfunction for the combination of a source and a sink.

A doublet is obtained as \( m \to \infty \), \( a \to 0 \) and \( \frac{ma}{\pi} \to K \).

Therefore, the streamfunction and velocity potential for a doublet can be expressed as:

For small values of \( a \):
\[
\arctan \left( \frac{2a r \sin \theta}{r^2 - a^2} \right) \approx \frac{2a r \sin \theta}{r^2 - a^2}
\]

Therefore:
\[
\psi = -\frac{m 2a r \sin \theta}{2\pi \left(r^2 - a^2\right)}
\]

As \( m \to \infty \), \( a \to 0 \) and \( \frac{ma}{\pi} \to K \):
\[
\psi = -\frac{K \sin \theta}{r} \quad \text{and} \quad \varnothing = \frac{K \cos \theta}{r}
\]

where \( K \) is the strength of the doublet.
Streamlines for a source-sink pair

Streamlines for a doublet