A pump delivers water at 3 ft³/s in the system below. The head loss in the pipe section is given by \(7.5 \frac{V^2}{2g}\), where \(V\) is the average speed in the pipe section. Find the power required to drive the pump if it is 80% efficient.

**Assumptions:**
- Steady flow
- Incompressible flow
- Uniform inlet and exit flow (\(\alpha_1 = \alpha_2 = 1\))
**Method 1 (standard method)**

The first step consists of finding the pressure along the surface 1. Let’s apply Bernoulli’s equation along a streamline between a point on the free surface and the point i at the midpoint of surface 2:

\[
\frac{p_0}{\rho} + \frac{V_0^2}{2} + gH = \frac{p_i}{\rho} + \frac{V_i^2}{2}
\]

We can assume that the velocity of the free surface (the rate at which the water level goes down) is negligible compared to the velocity at i: \( V_0 \ll V_i \). Therefore, the Bernoulli’s equation becomes:

\[
p_i = p_0 + \rho gH - \frac{1}{2} \rho \frac{V_i^2}{2}
\]

The application of the continuity equation to the CV yields:

\[
\bar{V}_1 = \bar{V}_2 = V
\]

Application of the control-volume energy balance to our CV yields:

\[
-\frac{\dot{W}_s}{\dot{m}g} = -\left( \frac{p}{\rho g} + z \right)_1 - \alpha_1 \frac{V_i^2}{2g} + \left( \frac{p}{\rho g} + z \right)_2 + \alpha_2 \frac{V_2^2}{2g} + H_L
\]

where \( H_L \) is the given head loss in the pipe section.

In order to write this energy balance, we have made use of the fact that \( p + \rho g z \) is constant across a pipe inlet (hydrostatic pressure distribution) and that \( \rho \bar{V}_1 A_1 = \rho \bar{V}_2 A_2 \).

If we now make the further assumptions:

\[
\left( \frac{p}{\rho g} + z \right)_1 \text{ is evaluated at } i
\]

\[
\left( \frac{p}{\rho g} + z \right)_2 \text{ is evaluated at } j
\]

\[
\alpha_1 = \alpha_2 \text{ (assume same velocity profile at 1 and 2)}
\]

and if we use the fact that \( \bar{V}_1 = \bar{V}_2 \), we can rewrite the last equation as:

\[
-\frac{\dot{W}_s}{\dot{m}g} = -\frac{p_i}{\rho g} + \frac{p_j}{\rho g} + (20 + H) + 7.5 \frac{V^2}{2g}
\]
The Bernoulli equation allows us to replace $p_i$, yielding:

$$\frac{-W_s}{\dot{m}g} = \frac{p_j - p_0}{\rho g} - H + \frac{V_i^2}{2g} + (20 + H) + 7.5 \frac{V^2}{2g}$$

Or, assuming $V_i = V_i = V$,

$$\frac{\dot{W}_S}{\dot{m}g} = \frac{p_j - p_0}{\rho g} + 20 + \frac{V^2}{2g} + 7.5 \frac{V^2}{2g}$$

Since the pump efficiency is 80%, the required power input is:

$$\text{power input} = \frac{\dot{W}_S / \dot{m}g}{0.8}$$

Therefore:

$$\text{power input} = \frac{1}{0.8} \left( \frac{p_j - p_0}{\rho g} + 20 + \frac{V^2}{2g} + 7.5 \frac{V^2}{2g} \right)$$
**Method 2 (more careful method)**

Let’s re-examine the problem without some of our earlier assumptions.

If we do not assume $\alpha_i = \alpha_2$:

$$\frac{\dot{W}_s}{mg} = \frac{p_j - p_i}{\rho g} + (20 + H) + (\alpha_2 - \alpha_1 + 7.5) \frac{V^2}{2g}$$

Substituting from the Bernoulli equation:

$$\frac{\dot{W}_s}{mg} = \frac{p_j - p_0}{\rho g} + \frac{V^2_{i}}{2g} + 20 + (\alpha_2 - \alpha_1 + 7.5) \frac{V^2}{2g}$$

$$\Leftrightarrow - \frac{\dot{W}_s}{mg} = \frac{p_j - p_0}{\rho g} + 20 + 7.5 \frac{V^2}{2g} + \frac{V^2_i}{2g} + (\alpha_2 - \alpha_1) \frac{V^2}{2g} - \frac{1}{2g} \left(V^2 - V^2_i\right)$$

The first box is our previous approximation result.
The second box contains additional terms.

In this equation, we have not assumed $V_i = \bar{V}_i = V$.

Let’s examine the additional terms in our “exact” result to see under what conditions they might be negligible.

- **Term** $\left(\alpha_2 - \alpha_1\right) \frac{V^2}{2g}$

Recall, $\alpha$ is a kinetic-energy correction factor defined by: $\alpha \bar{V}^3 = \bar{V}^3$

For a **uniform** flow, $\alpha = 1$. A turbulent-flow mean velocity profile has a shape like:

![Uniform Flow Profile](image1)

So, it is nearly uniform.

For a **parabolic** profile, $\alpha = 2$.

Therefore, there are cases for which $\alpha_2 - \alpha_1 = O(1)$, so that this term is not negligible.

- **Term** $\frac{1}{2g} \left(V^2 - V^2_i\right)$
Again, for uniform flow, \( V = V_i \) (the velocity along one streamline originating from the surface is equal to the average velocity along that surface). Therefore, for a uniform flow,

\[
\frac{1}{2g} (V^2 - V_i^2) = 0
\]

For turbulent flow, \( V \approx V_i \) (the velocity at one point on the surface is not very different from the average velocity on that surface). Therefore, for a turbulent flow,

\[
\frac{1}{2g} (V^2 - V_i^2) \approx 0
\]

For a parabolic profile,

\[
V = \frac{1}{2} V_i \Rightarrow V_i = 2V
\]

\[
\Rightarrow \frac{1}{2g} (V^2 - V_i^2) = \frac{1}{2g} (V^2 - 4V_i^2) = -\frac{3V^2}{2g}
\]

So, the term is not necessarily negligible.

The preceding discussion was provided to show that your assumptions should be questioned to see if they are reasonable for the problem at hand.

For this particular problem, 
\( V = Q/A = 61 \) ft/s

We decide whether pipe flow is laminar (parabolic profile) or turbulent (\( \sim \) uniform profile) by examining a dimensionless parameter called the Reynolds number:

\[
Re = \frac{Vd}{\nu}
\]

where:

- \( V \) is the characteristic velocity
- \( d \) is the pipe diameter
- \( \nu \) is the kinematic viscosity of the fluid

In this problem: \( Re = 1.4 \times 10^6 \).

Since this is well above the value \( Re = 2300 \) for transition to turbulent flow, we are justified in assuming nearly uniform profiles.

Therefore, using the equation in method 1:

\[
\text{power} = 628 \text{ ft} \quad \text{(expressed as a head)}
\]