ENERGY EQUATION: APPLICATION TO STEADY FLOW IN PIPES

General form of the energy equation:

Since the flow is steady, and since the pipe has one inlet and one outlet, the equation can be rewritten:

The study of the signs of the flux terms yields:
Assuming that enthalpy and potential energy are uniform over section 1 and over section 2:

Since the velocities at the inlet and outlet vary in the radial direction, the calculation of the remaining integrals should be carried out.

When studying the flow in a pipe, the average velocity over a cross section is generally considered.

The average of an arbitrary quantity $f$ over an area is given by:

$$
\bar{f} = \frac{1}{A} \int_A f \, dA
$$

The same definition applied to the velocity yields:

$$
\bar{V} = \frac{1}{A} \int_A V \, dA
$$

Also, similarly:

$$
\bar{V^3} = \frac{1}{A} \int_A V^3 \, dA
$$

With this definition, the energy equation becomes:

This expression can be further simplified by introducing the kinetic energy flux correction factor $\alpha$ defined as:

$$
\alpha = \frac{\bar{V^3}}{V^3}
$$
The equation can then be rewritten in terms of $\alpha$:

\[
\alpha_1 \frac{V_1}{A_1} = \alpha_2 \frac{V_2}{A_2},
\]

This expression can be further simplified by considering the conservation of mass applied to the CV, which results in the following equality:

\[
\rho A_1 V_1 = \rho A_2 V_2 = \dot{m}, \text{ where } \dot{m} \text{ is the mass flow rate}
\]

Substituting this relationship in the energy equation yields:

The shaft work in this example results from the work exerted by the flow on the turbine and the work exerted by the pump on the flow:

Therefore, the general energy equation for a steady flow through a pipe can be written: