CONTROL VOLUME ANALYSIS: EQUATION SUMMARY

DEFINITION OF VARIABLES
- In the following definitions, \( N \) is an arbitrary extensive property and \( \eta \) is \( N \) per unit mass:

\[
N = \iiint_{V(t)} \eta \rho dV
\]

- For moving CVs, the absolute \((\mathbf{V})\), relative \((\mathbf{W})\) and CV \((\mathbf{V}_{cv})\) velocities satisfy the following relation:

\[
\mathbf{V} = \mathbf{W} + \mathbf{V}_{cv}
\]

REYNOLDS TRANSPORT THEOREM

\[
\left( \frac{dN}{dt} \right)_{\text{system}} = \frac{\partial}{\partial t} \iiint_{CV} \eta \rho dV + \iint_{CS} \eta \rho \mathbf{V} \cdot dA
\]

Time rate of change of \( N \) in the system at time \( t \) = Time rate of change of \( N \) in the CV at time \( t \) + Net efflux of \( N \) (or rate at which \( N \) leaves the CV) though the CS at time \( t \)

CONSERVATION OF MASS (CONTINUITY EQUATION)

\[
\left( \frac{dM}{dt} \right)_{\text{system}} = \frac{\partial}{\partial t} \iiint_{CV} \rho dV + \iint_{CS} \rho \mathbf{V} \cdot dA = 0
\]

Time rate of change of \( M \) in the system at time \( t \) = Time rate of change of \( M \) in the CV at time \( t \) + Net efflux of \( M \) (or rate at which \( M \) leaves the CV) though the CS at time \( t \) = 0

For moving CV:

\[
\frac{\partial}{\partial t} \iiint_{CV} \rho dV + \iint_{CS} \rho \mathbf{W} \cdot dA = 0
\]
**Balance of Linear Momentum**

\[
\left( \frac{d(MV)}{dt} \right)_{\text{system}} = \frac{\partial}{\partial t} \iiint_{\text{CV}} \rho V dV + \iint_{\text{CS}} (\rho \vec{V}) \cdot dA = \sum F_{\text{on CV}}
\]

Time rate of change of linear momentum in the system at time \( t \) = Time rate of change of linear momentum in the CV at time \( t \) + Net efflux of linear momentum (or rate at which linear momentum leaves the CV) though the CS at time \( t \) = Sum of the forces exerted on the CV at time \( t \)

For moving CV:

\[
\frac{\partial}{\partial t} \iiint_{\text{CV}} \rho V dV + \iint_{\text{CS}} (\rho \vec{V}) \cdot dA = \sum F_{\text{on CV}}
\]

**Balance of Moment-of-Momentum**

\[
\left( \frac{d(r \times MV)}{dt} \right)_{\text{system}} = \frac{\partial}{\partial t} \iiint_{\text{CV}} r \times \rho V dV + \iint_{\text{CS}} (r \times \rho \vec{V}) \cdot dA = \sum M_{\text{on CV}}
\]

Time rate of change of moment-of-momentum in the system at time \( t \) = Time rate of change of moment-of-momentum in the CV at time \( t \) (here, \( V \) is the absolute velocity) + Net efflux of moment-of-momentum (or rate at which moment-of-momentum leaves the CV) though the CS at time \( t \) (in \( V \cdot dA \), \( V \) is the relative fluid velocity with respect to the CS) = Sum of the moments exerted on the CV at time \( t \)
ENERGY EQUATION

\[
\left( \frac{d\mathcal{E}}{dt} \right)_{\text{system}} = \frac{\partial}{\partial t} \int \int \int_{\text{CV}} \rho \left( \frac{V^2}{2} + gz + u \right) d\mathcal{V} + \int \int_{\text{CS}} \rho \left( \frac{V^2}{2} + gz + u \right) V \cdot d\mathcal{A} = \dot{Q} - \dot{W}
\]

Time rate of change in stored energy in the system at time \( t \) = Time rate of change in stored energy in the CV at time \( t \) + Net efflux of energy through the CS at time \( t \) = Rate of heat addition to the fluid in the CV at time \( t \) + Rate of work done by the fluid in the CV at time \( t \)

Also written:

\[
\frac{\partial}{\partial t} \int \int \int_{\text{CV}} \rho \left( \frac{V^2}{2} + gz + u \right) d\mathcal{V} + \int \int_{\text{CS}} \rho \left( \frac{V^2}{2} + gz + h \right) V \cdot d\mathcal{A} = \dot{Q} - \dot{W}_s
\]