MEASURES OF FLUID COMPRESSIBILITY

BULK MODULUS
In an analogous manner to Young’s modulus of elasticity for solids, we can define a coefficient of compressibility \( E_v \) (also called the bulk modulus) for fluids as:

\[
E_v = -\frac{dP}{dV/V} = \frac{dP}{d\rho/\rho}
\]

units: \([E_v] = \text{Pa}\)

\(dP\) is the differential change in pressure needed to create a differential change in volume \(dV\) of a volume \(V\).

Differential form:

\[
E_v = \rho \left( \frac{\partial P}{\partial \rho} \right)_T
\]

For an ideal gas:

\(P = \rho RT\), and \(E_v = P\).

ISOTHERMAL COMPRESSIBILITY
The inverse of the bulk modulus is the isothermal compressibility \(\alpha\):

\[
\alpha = \frac{1}{E_v} = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial P} \right)_T
\]

units: \([\alpha] = 1/\text{Pa}\)

COEFFICIENT OF VOLUME EXPANSION
The density of a fluid depends generally more strongly on temperature than on pressure. This dependence is quantified by the coefficient of volume expansion \(\beta\):

\[
\beta = \frac{\Delta V/V}{\Delta T} = -\frac{\Delta \rho/\rho}{\Delta T}
\]

units: \([\beta] = 1/\text{K}\)

Differential form:

\[
\beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_P
\]

For an ideal gas:

\(P = \rho RT\), and \(\beta = \frac{1}{T}\).

MACH NUMBER
From thermo, for an ideal gas:

\[
c^2 = k \left( \frac{\partial P}{\partial \rho} \right)_T = kRT,
\]

where \(c\) is the speed of sound and \(k\) is the specific heat ratio.

The Mach number \(Ma\) is an important parameter in the analysis of compressible fluid flows and is defined as the ratio of the actual speed of the fluid \(V\) to the speed of sound in the same fluid \(c\) (\(c_{\text{air}} = 340\text{m/s}\)):

\[
Ma = \frac{V}{c}
\]

unit: dimensionless

The compressibility of gases is generally not important until \(Ma > 0.3\).