Name/UID: __________________________

Format:
- This exam consists of three independent problems:
  o Problem 1: 9 points
  o Problem 2: 21 points
  o Problem 3: 20 points
- There is one optional bonus question at the end of problem 2 (2 points)
- One appendix is provided at the end of this document: Saturated water property tables

Policies:
- **Material permitted:** class notes, calculators, handouts, homework assignments and solutions
- **Material not permitted:** textbooks, tablets, laptops, phones
- All assumptions must be justified to get the maximum number of points for each question.
- Clearly indicate the final answer/result by underlining it or drawing a box around it.
- Place your name/UID in the top, right-hand corner of each page.
- Use only black/blue ink.

Honor code:
“As a member of the Wright State University community, I will not participate in or tolerate academic dishonesty.”

Signature: __________________________

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
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</table>

Comments:
PROBLEM 1: CARNOT CYCLE (9 pts)

A solar collector consisting of multiple solar panels provides thermal energy \( \dot{Q}_c \) at a temperature \( T_c = 80^\circ C \) to a reversible Carnot heat engine. The flux of solar energy striking the solar collector is \( \dot{q}_{sun} = 1 \text{ kW/m}^2 \).

The engine rejects heat to the surroundings at \( T_{sur} = 25^\circ C \) and generates a power output \( \dot{W} = 5 \text{ kW} \).

In this problem, we would like to determine the number of solar panels required to achieve the desired power output.

a) (2 pts) Calculate the thermal efficiency \( \eta_{th} \) of the Carnot heat engine.

\[
\eta_{th} = 1 - \frac{T_c}{T_H} = 1 - \frac{25 + 273}{80 + 273} = 0.156
\]

\[
\eta_{th} = 15.6\%
\]

b) (2 pts) Using your result from part a), calculate the thermal energy \( \dot{Q}_c \) required to generate the desired power output.

\[
\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}}{\dot{Q}_c} \Rightarrow \dot{Q}_c = \frac{\dot{W}}{\eta_{th}} = \frac{5 \times 10^3}{0.156} = 31.7 \text{ kW}
\]

\[
\dot{Q}_c = 31.7 \text{ kW}
\]

c) (5 pts) If each solar panel has a surface area \( A_p = 1 \text{ m}^2 \) and converts \( \varepsilon = 50\% \) of the incident solar energy into usable thermal energy, develop an expression for the number of solar panels \( N \) required to achieve the desired power output in terms of \( (A_p, \varepsilon, \dot{Q}_c, \dot{q}_{sun}) \). Calculate \( N \) numerically.

- Power generated by each solar panel: \( \dot{Q}_p = \varepsilon \dot{q}_{sun} A_p \)
- Power generated by \( N \) panels: \( N \varepsilon \dot{q}_{sun} A_p \)

\[
N \varepsilon \dot{q}_{sun} A_p = \dot{Q}_c
\]

Analytical expression: \( N = \frac{\dot{Q}_c}{\varepsilon \dot{q}_{sun} A_p} \)

Numerical application: \( N = 65 \)
**Problem 2: Entropy and Properties of Pure Substances (21 pts)**

A closed container (volume: \( V = 2 \text{ m}^3 \)) contains a mass \( m = 4 \text{ kg} \) of water at a pressure \( P = 200 \text{ kPa} \). The surroundings are maintained at a temperature \( T_{\text{sur}} = 160^\circ \text{C} \). Heat is transferred to the vessel until the water is all just evaporated (saturated vapor state). Assume no change in potential and kinetic energy.

\[
\begin{align*}
P_1 &= 200 \text{ kPa} \\
V &= 2 \text{ m}^3 \\
T_{\text{sur}} &= 160^\circ \text{C}
\end{align*}
\]

\[
\text{surroundings} \\
\text{water (2-phase mixture)} \\
\quad \quad \Downarrow Q_{i,2} \\
\text{water (saturated vapor)} \\
\text{surroundings}
\]

\[
\begin{align*}
P_{\text{sat}} &= 200 \text{ kPa} \\
V_{\text{sat}} &= 2 \text{ m}^3 \\
\end{align*}
\]

---

**a)** (6 pts) Using the water property table, calculate the quality \( x_1 \) of the 2-phase mixture at state 1 and determine the specific entropy \( s_1 \) and the specific internal energy \( u_1 \). Show all numerical values used in the calculations.

- **Specific volume:** \( v_1 = \frac{V}{m} = \frac{2}{4} = 0.5 \text{ m}^3/\text{kg} \).
  \[v_1 = v_{f1} + x_1 v_{fg1} \Rightarrow x_1 = \frac{v_1 - v_{f1}}{v_{fg1}} = \frac{0.5 - 0.5}{0.88578 - 0.5} \]
- **Entropy:** \( s_1 = s_{f1} + x_1 s_{fg1} = 1.5302 + 0.564 \times 5.5968 \)
- **Internal energy:** \( u_1 = u_{f1} + x_1 u_{fg1} = 509.5 + 0.564 \times 2024.6 \)

\[
\begin{align*}
x_1 &= 0.564 \\
s_1 &= 4.6868 \text{ kJ/}^\circ \text{C.kg} \\
u_1 &= 1644.8 \text{ kJ/} \text{kg}
\end{align*}
\]

---

**b)** (6 pts) Using the water property table, calculate the final pressure \( P_2 \), the final specific entropy \( s_2 \), and the final specific internal energy \( u_2 \). If interpolation is needed, formulate the equation to be used to obtain each missing property.

**Constant volume process** \( v_2 = v_1 = 0.5 \text{ m}^3/\text{kg} \).
**Find specific volume:** \( v_2 = v_{fg2} \) (saturated vapor)

\[
\Rightarrow \text{From the table: } v_g = 0.49133 < v_2 = 0.5 < v_g = 0.52422 \text{ m}^3/\text{kg}.
\]
c) **(2 pts)** Calculate the entropy change of the water \( \Delta S_{\text{water}} \) in the vessel during the heat transfer process.

\[
\Delta S_{\text{water}} = m \left( s_2 - s_1 \right) = 4 \left( 6.9232 - 4.6868 \right)
\]

\[
\Delta S_{\text{water}} = 8.3456 \text{ J/K}
\]

d) **(7 pts)** Using the 1st law, determine the heat transfer \( Q_{12} \) added to the vessel and calculate the entropy change for the surroundings \( \Delta S_{\text{sur}} \).

- 1st law, closed system: \( \Delta E = \Delta U = Q_{12} - W_{\text{cyc}} \)
  
  \[
  Q_{12} = m \left( u_2 - u_1 \right) = 4 \left( 2550.3 - 1646.4 \right)
  \]

- \( \Delta S_{\text{sur}} = \frac{Q_{12} \cdot \Delta T_{\text{sur}}}{T_{\text{sur}}} = -\frac{Q_{12}}{T_{\text{sur}}} = -\frac{3615.6}{160 + 273} \)

\[
Q_{12} = 3615.6 \text{ J}
\]

\[
\Delta S_{\text{sur}} = -83501 \text{ J/K}
\]

**Bonus question:**

e) **(2 pts)** Using your results from part c) and part d), develop an expression for the entropy \( S_{\text{gen}} \) generated during this heat transfer process and calculate numerically.

**Analytical expression:**

\[
S_{\text{gen}} = \Delta S_{\text{water}} + \Delta S_{\text{sur}}
\]

**Numerical application:**

\[
S_{\text{gen}} = 0.59 \text{ J/K}
\]
**Problem 3: Power Cycle of an Ideal Gas (20 pts)**

An ideal gas is contained in a piston-cylinder device and undergoes a power cycle as follows:

<table>
<thead>
<tr>
<th>Process</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1→2</td>
<td>Isentropic compression from $T_1 = 20^\circ$C with a compression ratio $r = 5$</td>
</tr>
<tr>
<td>2→3</td>
<td>Constant pressure heat addition</td>
</tr>
<tr>
<td>3→1</td>
<td>Constant volume heat rejection ($w_{31} = 0$ and $q_{31} = -1840.3$ kJ/kg)</td>
</tr>
</tbody>
</table>

The gas has constant properties ($c_v = 0.7$ kJ/kg·K, $c_p = 1.0$ kJ/kg·K, $k = c_p / c_v = 1.429$, $R = 0.3$ kJ/kg·K). Assume no change in potential and kinetic energy throughout the cycle.

a) (6 pts) Sketch the cycle in the $P - v$ and $T - s$ diagrams.

![P-v and T-s diagrams](image_url)

b) (2 pts) Develop an expression for the temperature $T_2$ at the end of the isentropic process in terms of $(T_1, r, k)$, and calculate numerically. Hint: for an isentropic process of an ideal gas with constant specific heats: $T v^{k-1} = \text{constant}$ and $T P^k = \text{constant})$

*Isentropic: $T_1 \frac{v_1^{k-1}}{v_2^{k-1}} = T_2$ $\Rightarrow T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{k-1}$

Analytical expression: $T_2 = T_1 r^{k-1}$

Numerical application: $T_2 = 584.4$ K

c) (3 pts) Using the 1st law, calculate the work $w_{12}$ and the heat transfer $q_{12}$ during the isentropic process.

1st law, closed system: $u_2 - u_1 = q_{12} - w_{12}$

$\Rightarrow w_{12} = u_1 - u_2 = c_v (T_1 - T_2)$ (adiabatic)

$w_{12} = -203.98$ kJ/kg

$q_{12} = 0$
d) \(2.5\) pts Develop an expression for the temperature \(T_3\) in terms of \((T_2, r)\), and calculate numerically.

\[
\text{Ideal gas: } \frac{P_2 u_2}{T_2} = \frac{P_3 u_3}{T_3} \quad \text{Since } P_2 = P_3, \quad \frac{u_2}{T_2} = \frac{u_3}{T_3}
\]

\[
\Rightarrow \quad T_3 = T_2 \frac{u_3}{u_2} = T_2 \frac{u_1}{u_2}
\]

Analytical expression: \(T_3 = T_2 \frac{u_1}{u_2}\)

Numerical application: \(T_3 = 2922 \text{ K}\)

e) \(4\) pts Develop expressions for the work \(w_{23}(R, T_2, T_3)\) and the heat transfer \(q_{23}(c_v, R, T_2, T_3)\), and calculate numerically.

1st law during \(2 \rightarrow 3\):

\[
w_{23} = u_3 - u_2 = q_{23} - w_{23}
\]

\[
w_{23} = P \int_{u_2}^{u_3} dv = P (u_3 - u_2).
\quad \text{For ideal gas: } P u = R T
\]

\[
\Rightarrow \quad w_{23} = R (T_3 - T_2)
\]

1st law:

\[
q_{23} = (u_3 - u_2) + w_{23} = c_v (T_3 - T_2) + R (T_3 - T_2)
\]

\[
\Rightarrow \quad q_{23} = c_v (T_3 - T_2) = (c_v + R) (T_3 - T_2)
\]

Analytical expressions:

\[
w_{23} = R (T_3 - T_2)
\]

\[
q_{23} = (R + c_v) (T_3 - T_2)
\]

Numerical applications:

\[
w_{23} = 701.28 \text{ kJ/kg}
\]

\[
q_{23} = 2337.6 \text{ kJ/kg}
\]

f) \(2.5\) pts Using your results and the information given during process \(3 \rightarrow 1\) \((w_{31} = 0\) and \(q_{31} = -1840.3 \text{ kJ/kg}\)), calculate the thermal efficiency of this cycle \(\eta_{th}\).

\[
\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{12} + w_{23} + w_{31}}{q_{23}}
\]

\[
\Rightarrow \quad \eta_{th} = \frac{-203.98 + 701.28}{2337.6}
\]

\[
\eta_{th} = 21.3\%
\]
# APPENDIX: Saturated Water Properties

<table>
<thead>
<tr>
<th>Press., temp., P kPa</th>
<th>Specific volume, m³/kg</th>
<th>Internal energy, kJ/kg</th>
<th>Enthalpy, kJ/kg</th>
<th>Entropy, kJ/kg·K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sat. liquid, v&lt;sub&gt;r&lt;/sub&gt;</td>
<td>Sat. vapor, v&lt;sub&gt;v&lt;/sub&gt;</td>
<td>Sat. liquid, u, u&lt;sub&gt;rg&lt;/sub&gt;</td>
<td>Sat. vapor, u&lt;sub&gt;v&lt;/sub&gt;</td>
</tr>
<tr>
<td>175</td>
<td>116.04</td>
<td>0.001057</td>
<td>1.0037</td>
<td>486.82</td>
</tr>
<tr>
<td>200</td>
<td>120.21</td>
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<td>0.89578</td>
<td>504.50</td>
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<tr>
<td>225</td>
<td>123.97</td>
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<td>0.79329</td>
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<td>250</td>
<td>127.41</td>
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<td>275</td>
<td>130.58</td>
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<td>0.65732</td>
<td>548.57</td>
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<tr>
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<td>133.52</td>
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<td>0.60582</td>
<td>561.11</td>
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<tr>
<td>325</td>
<td>136.27</td>
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<td>572.84</td>
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<tr>
<td>350</td>
<td>138.86</td>
<td>0.001079</td>
<td>0.52422</td>
<td>583.89</td>
</tr>
<tr>
<td>375</td>
<td>141.30</td>
<td>0.001081</td>
<td>0.49133</td>
<td>594.32</td>
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<tr>
<td>400</td>
<td>143.61</td>
<td>0.001084</td>
<td>0.46242</td>
<td>604.22</td>
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<tr>
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<td>147.90</td>
<td>0.001088</td>
<td>0.41392</td>
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<tr>
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<td>151.83</td>
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<td>155.46</td>
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<td>0.31560</td>
<td>669.72</td>
</tr>
<tr>
<td>650</td>
<td>161.98</td>
<td>0.001104</td>
<td>0.29260</td>
<td>683.37</td>
</tr>
</tbody>
</table>