Problem 1

1- Velocity calculation

Control volume definition: volume of water contained with the pump

Since the question is about the calculation of inlet and outlet velocities, we consider the conservation of mass principle.

General expression:

\[ \frac{\partial m_{cv}}{\partial t} - \dot{m}_1 + \dot{m}_2 = 0 \]

Assumptions:
- the pump operates steadily \( \Rightarrow \frac{\partial}{\partial t} \left( \right) = 0 \)
- the properties of the fluid and the flow are uniform over the inlet and over the outlet

Reduced equation:

\[ 0 - \dot{m}_1 + \dot{m}_2 = 0 \]

\[ \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m} \]

We then expand the expression of the mass flow rate:

\[ \int_1 \rho V dA = \int_2 \rho V dA = \dot{m} \]

Since all properties and characteristics are uniform over the inlet and the outlet, the integrals can be calculated as:

\[ \rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \dot{m} \]

\[ \Rightarrow \rho_1 V_1 \left( \pi \frac{d_1^2}{4} \right) = \rho_2 V_2 \left( \pi \frac{d_2^2}{4} \right) = \dot{m} \]

Inlet velocity calculation: \( V_1 = \frac{4\dot{m}}{\pi \rho_1 d_1^2} \)

The water inlet properties are: \( P_1 = 100 \text{ kPa} \) and \( T_1 = 15^\circ \text{C} \). Using the \( T - \nu \) diagram, this point falls in the compressed liquid region since: \( T_1 < T_{sat} \) @ 100 kPa
Instead of using the compressed liquid table (Table A-7), which does not include properties at a pressure of 100 kPa, we approximate the compressed liquid properties as the saturated liquid properties from the saturated water table (Table A-4).

@ \( T = 15^\circ C \): \( \nu_j = \nu_i = 0.001001 \text{ m}^3/\text{kg} \)

**Numerical application:**

\[
V_1 = \frac{4 \times 0.5}{\pi \times (0.001001)^{-1} \times (0.01)^2} \Rightarrow V_1 = 6.37 \text{ m/s}
\]

**Outlet velocity calculation:**

\[
V_2 = \frac{4 \dot{m}}{\pi \rho_2 d_2^2}
\]

Water being treated as an incompressible fluid in the pump: \( \rho_1 = \rho_2 = \frac{1}{\nu_i} \)

**Numerical application:**

\[
V_2 = \frac{4 \times 0.5}{\pi \times (0.001001)^{-1} \times (0.015)^2} \Rightarrow V_2 = 2.83 \text{ m/s}
\]

2- **Condition calculation with increased inlet temperature**

When the inlet temperature is \( T_i = 40^\circ C \), the specific volume of saturated liquid water becomes: \( \nu_i = 0.001008 \text{ m}^3/\text{kg} \)

Therefore:

\[
V_1 = \frac{4 \times 0.5}{\pi \times (0.001008)^{-1} \times (0.01)^2} \Rightarrow V_1 = 6.42 \text{ m/s}
\]

\[
V_2 = \frac{4 \times 0.5}{\pi \times (0.001008)^{-1} \times (0.015)^2} \Rightarrow V_2 = 2.85 \text{ m/s}
\]

We conclude that an increase in inlet water temperature from 15°C to 40°C is associated with a 0.78% change in velocity at the inlet and outlet.

**Problem 2**

1- **Mass flow rate and exit velocity calculation**

The mass flow rate of the steam \( \dot{m} \) is related to the rate at which the volume of water changes over time \( \dot{V} \):

\[
\dot{m} = \rho_{\text{steam}} A \dot{V}_{\text{steam}} = \rho_{\text{water}} \dot{V}
\]
The density of the steam can be obtained from the saturated water table at a pressure \( P = 138 \text{ kPa} \). Since the table only provides the specific volume of vapor at 125 and 150 kPa, we use interpolation to obtain the specific volume at 138 kPa:

\[
\nu_{g,138 \text{ kPa}} = \nu_{g,125 \text{ kPa}} + \frac{138 - 125}{150 - 125} (\nu_{g,150 \text{ kPa}} - \nu_{g,125 \text{ kPa}})
\]

\[
\Rightarrow \nu_{g,138 \text{ kPa}} = 1.375 + \frac{138 - 125}{150 - 125} (1.1594 - 1.375)
\]

\[
\Rightarrow \nu_{g,138 \text{ kPa}} = 1.2629 \text{ m}^3/\text{kg}
\]

Similarly, for the specific volume of the water:

\[
\nu_{f,138 \text{ kPa}} = \nu_{f,125 \text{ kPa}} + \frac{138 - 125}{150 - 125} (\nu_{f,150 \text{ kPa}} - \nu_{f,125 \text{ kPa}})
\]

\[
\Rightarrow \nu_{f,138 \text{ kPa}} = 0.001048 + \frac{138 - 125}{150 - 125} (0.001053 - 0.001048)
\]

\[
\Rightarrow \nu_{f,138 \text{ kPa}} = 0.0010506 \text{ m}^3/\text{kg}
\]

Therefore:

\[
\dot{m} = \rho_{\text{water}} \dot{V} = \rho_{\text{water}} \frac{V}{\Delta t}
\]

**Numerical application:**

\[
\dot{m} = (0.0010506)^{-1} \times \frac{2.27 \times 10^{-3}}{45 \times 60} \Rightarrow \dot{m} = 0.80025 \times 10^{-3} \text{ kg/s}
\]

We have established that \( \dot{m} = \rho_{\text{steam}} AV_{\text{steam}} \), therefore:

\[
V_{\text{steam}} = \frac{\dot{m}}{\rho_{\text{steam}} A}
\]

**Numerical application:**

\[
V_{\text{steam}} = \frac{0.80025 \times 10^{-3}}{(1.2629)^{-1} \times 10^{-3}} \Rightarrow V_{\text{steam}} = 10.106 \text{ m/s}
\]
2- **Total energy of exiting steam**

The total energy $e_{out}$ is calculated as the sum of specific enthalpy, specific kinetic energy and specific potential energy at the outlet:

$$e_{out} = h_{steam} + \frac{V_{steam}^2}{2} + g Z_{steam}$$

Neglecting potential and kinetic energy:

$$e_{out} = h_{steam}$$

The specific enthalpy is obtained by interpolation in the saturated water table:

$$h_{g,138 \text{ kPa}} = h_{g,125 \text{ kPa}} + \frac{138 - 125}{150 - 125} \left( h_{g,150 \text{ kPa}} - h_{g,125 \text{ kPa}} \right)$$

$$\Rightarrow h_{g,138 \text{ kPa}} = 2684.9 + \frac{138 - 125}{150 - 125} \left( 2693.1 - 2684.9 \right)$$

$$\Rightarrow e_{out} = h_{g,138 \text{ kPa}} = 2689.16 \text{ kJ/kg}$$

3- **Total rate of energy exiting**

The total rate of energy $\dot{E}_{out}$ is calculated as:

$$\dot{E}_{out} = \dot{m} e_{out}$$

**Numerical application:**

$$\dot{E}_{out} = \left( 0.80025 \times 10^{-3} \right) \times 2689.16 \Rightarrow \dot{E}_{out} = 2.152 \text{ kJ/s}$$

### Problem 3

1- **Exit velocity calculation**

**Control volume definition:** volume of steam contained within the nozzle

**General expression of energy equation in CV form:**

$$\frac{\partial E_{CV}}{\partial t} - \dot{m}_1 \left( h_1 + \frac{V_1^2}{2} + g z_1 \right) + \dot{m}_2 \left( h_2 + \frac{V_2^2}{2} + g z_2 \right) = \dot{Q} - \dot{W}_s$$
Assumptions:
- the nozzle operates steadily \( \frac{\partial}{\partial t} ( ) = 0 \)
- the properties of the fluid and the flow are uniform over the inlet and over the outlet
- there is no shaft work in this device (no work is added to the CV, no work is extracted from the CV) \( \dot{W}_s = 0 \)
- the nozzle is not adiabatic and a heat loss at a rate \( \dot{Q}_{\text{loss}} \) is exiting the CV
- the inlet and the outlet are at the same elevation \( z_1 = z_2 \)

Reduced energy equation:

\[-m_1 \left( h_1 + \frac{V_1^2}{2} + gz_1 \right) + m_2 \left( h_2 + \frac{V_2^2}{2} + gz_2 \right) = \dot{Q} = -\dot{Q}_{\text{loss}} \]

The application of the conservation of mass to the same CV yields: \( m_1 = m_2 = \dot{m} \)

Therefore, the reduced energy equation can be rewritten:

\[-\dot{m} \left( h_1 + \frac{V_1^2}{2} \right) + \dot{m} \left( h_2 + \frac{V_2^2}{2} \right) = -\dot{Q}_{\text{loss}} \]

\[ \Rightarrow \dot{m} \left( h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \right) = -\dot{Q}_{\text{loss}} \]

\[ \Rightarrow V_2 = \sqrt{V_1^2 + 2 \left( h_1 - h_2 + \frac{-Q_{\text{loss}}}{\dot{m}} \right)} \]

Inlet steam properties:
At the inlet: \( T_1 = 400 ^\circ \text{C} \) and \( P_1 = 800 \text{ kPa} \)

We can verify that the steam is superheated by looking at the saturation temperature at a pressure of 800 kPa (Table A-5): \( T_{\text{sat}} @ 800 \text{ kPa} = 170.41 ^\circ \text{C} \). Since \( T_1 > T_{\text{sat}} @ 800 \text{ kPa} \), the steam is superheated. Therefore, all properties are obtained from Table A-6.

We find: \( h_1 = 3267.7 \text{ kJ/kg} \) and \( \nu_1 = 0.38429 \text{ m}^3/\text{kg} \)

Outlet steam properties:
At the inlet: \( T_1 = 400 ^\circ \text{C} \) and \( P_1 = 800 \text{ kPa} \)
We can verify that the steam is superheated by looking at the saturation temperature at a pressure of 200 kPa (Table A-5): \( T_{\text{sat}} @ 200 \text{ kPa} = 120.21^\circ \text{C} \). Since \( T_2 > T_{\text{sat}} @ 200 \text{ kPa} \), the steam is superheated. Therefore, all properties are obtained from Table A-6.

We find: \( h_2 = 3072.1 \text{ kJ/kg} \) and \( \nu_2 = 1.31623 \text{ m}^3/\text{kg} \)

**Numerical application:**

\[
V_2 = \sqrt{10^2 + 2\left(3267.7 - 3072.1 - \frac{25}{(0.38429)^{-1} \times 10 \times (800 \times 10^{-4})}\right) \times 10^3}
\]

\[\Rightarrow V_2 = 606 \text{ m/s}\]

2- Exit volume flow rate calculation

The volume flow rate at the outlet is:

\[
\dot{V}_2 = \frac{m_2}{\rho_2} = \rho_1 V_1 A_1
\]

**Numerical application:**

\[
\dot{V}_2 = \frac{(0.38429)^{-1} \times 10 \times (800 \times 10^{-4})}{(1.31623)^{-1}} \Rightarrow \dot{V}_2 = 2.74 \text{ m}^3
\]