Problem 1
The resistance heater consumes electric energy at a rate $P = 4 \text{ kW} = 4 \text{ kJ/s}$. Therefore, the total amount of electric energy $\mathcal{E}$ used during the time period $t$ is:

$$\mathcal{E} = Pt$$

Numerical application: $E = 4 \times 3 = 12 \text{ kWh}$

To convert this result in kJ, we note that $1 \text{ kWh} = (1 \text{ kJ/s})(3600 \text{ s}) = 3600 \text{ kJ}$

Therefore:

$$E = 12 \text{ kWh} = 12 \times 3600 = 43,200 \text{ kJ}$$

Problem 2
The pool volume ($V$) depends on the filling time ($t$), the hose cross-sectional area ($\pi d^2/4$) and the flow velocity ($V$):

$$V = V(t, \pi d^2/4, V)$$

Let’s examine the units of each variable:

$$[V] = m^3 \quad [t] = s \quad [\pi d^2/4] = m^2 \quad [V] = m \cdot s^{-1}$$

The only combination possible for $t$, $\pi d^2/4$ and $V$ to yield a quantity consistent with time is by taking their product:

$$t(\pi d^2/4)V, \text{ since } [t(\pi d^2/4)V] = s \cdot m^2 \cdot \frac{m}{s} = m^3$$

Therefore, we conclude that:

$$V = \frac{t\pi d^2V}{4}$$
Problem 3
Relation between density and elevation:

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Using Excel, the data can be fitted with a third-order polynomial to yield a correlation coefficient $R^2 = 0.99$:

$$\rho = -3.932 \times 10^{-14} h^3 + 3.657 \times 10^{-9} h^2 - 1.141 \times 10^{-4} h + 1.222$$

Using this approximation, the density at an elevation of 7000 m is: $\rho = 0.589 \text{ kg/m}^3$

We will calculate the mass of the atmosphere by calculating the mass of an infinitesimal spherical shell of radius $r$ and thickness $dr$, and by integrating this mass over of the entire atmospheric layer.

The volume of a thin spherical shell is: $dV = 4\pi r^2 dr$. Its mass is: $dm = 4\pi r^2 \rho dr$.

The mass of the atmosphere can be calculated by integrating the mass of the shell over the thickness of the atmosphere:

$$m_{\text{atmosphere}} = \int_{r=R}^{r=R_{\text{gw}}} 4\pi r^2 \rho dr$$
Using the correlation found earlier:

\[ m_{\text{atmosphere}} = \int_{r=R}^{r=R_m} 4\pi r^2 \left( -3.932 \times 10^{-14} h^3 + 3.657 \times 10^{-9} h^2 - 1.141 \times 10^{-4} h + 1.222 \right) dr \]

Switching all variables to \( h \):

\[ m_{\text{atmosphere}} = 4\pi \int_{h=0}^{h=R_m} (R + h)^2 \left( -3.932 \times 10^{-14} h^3 + 3.657 \times 10^{-9} h^2 - 1.141 \times 10^{-4} h + 1.222 \right) dh \]

The calculation yields: \( m_{\text{atmosphere}} = 5.17 \times 10^{19} \text{kg} \)