Numerical Simulation of Microfiltration of Oil-in-Water Emulsions

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Outline

• Introduction

• Results
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    • Oil film entry into pores of arbitrary shape (effect of transmembrane pressure).
    • Oil drop entry dynamics into circular pores under shear flow (effect of shear rate and transmembrane pressure).
  • Part 2:
    • Effect of confinement on drop dynamics.
    • Effect of viscosity ratio on drop dynamics.
    • Effect of surface tension coefficient on drop dynamics.
    • Effect of contact angle on drop dynamics.
    • Effect of drop size on drop dynamics.

• Conclusions

• Proposed future work
Microfluidics: Characteristics and Applications

Fluid flow at micron-scales.

- Laminar flow (small Re)
- High surface to volume ratio.
- Negligible gravity and inertia.
- Encapsulation of molecules, bio-reagents, and cells for delivery or reaction.
- Flow through porous media (e.g. oil extraction)
- Biological flows (e.g. flow inside blood vessels)

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Emulsions: Properties and Applications

Mixture of two immiscible fluids

- Dispersed droplets in continuous phase.

- Various forms of Immiscible fluids
  - i. Coalescence
  - ii. Floculation
  - iii. Creaming
  - iv. Breaking

- Beneficial emulsion production: food (left) & oil transportation (right).

- Unfavorable emulsion production: industrial wastewater (left) & biodiesel washing (right).

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Emulsions: Production and Separation

- Mixture of two-immiscible fluid

- Ultrasound emulsification
  - Continuous phase
  - Dispersed phase

- Mechanical agitation
  - Membrane

- Membrane emulsification

- Chemical Demulsification
- Gravity separator
- Membrane microfiltration

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Microfiltration of oil: Motivations and Methods

- Oil spill in the Gulf of Mexico.
- Industrial wastewater in Thailand.
- Cutting fluids for machining.

### Oil-Water Separation Methods

<table>
<thead>
<tr>
<th>Centrifuge separation</th>
<th>Membrane filtration</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Centrifuge" /></td>
<td><img src="image2.png" alt="Membrane" /></td>
</tr>
</tbody>
</table>

**Centrifuge separation**

- **Hydrocyclone**

- ![Hydrocyclone](image3.png)

**Membrane filtration**

- **Dead-End**

- ![Dead-End](image4.png)

- **Crossflow**

- ![Crossflow](image5.png)
Membrane Fouling, Permeate Flux, and Pore Shape

- Optimum design: high water flux and high oil rejection.
- Major issue: Fouling reduces water flux in time.

Circular Pores

Slotted Pores

- Slotted pores: higher flux rates than circular pores due to lower fouling rate.

Complete Blocking: happens at the initial stages of filtration. A droplet blocks the pore completely.

Standard blocking: deposition of drops inside the pore.

Cake Formation: happens at the final stages of filtration. A layer of drops forms on the surface.

Flux decline in crossflow microfiltration.

- Circular Pores: deposition of drops inside the pore.
- Slotted Pores: higher flux rates than circular pores due to lower fouling rate.

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Critical Pressure of Permeation

Young-Laplace Equation: pressure drop across an interface:

$$\Delta P = \sigma \cos \theta \left( \frac{1}{R_x} + \frac{1}{R_y} \right)$$

For a spherical interface:

$$\Delta P = 2\sigma \cos \theta \left( \frac{1}{R_d} \right)$$

Critical pressure for a droplet deposited on a circular pore:

$$P_{\text{crit}} = 2\sigma \cos(\theta) \frac{\cos(\theta)}{r_{\text{pore}}} \left[ 2 + 3 \cos \theta - \cos^3 \theta \right]^{1/3} \left[ \frac{4 \left( \frac{r_{\text{drop}}}{r_{\text{pore}}} \right)^3 \cos^3 \theta - (2 - 3 \sin \theta + \sin^3 \theta)}{4 \left( \frac{r_{\text{drop}}}{r_{\text{pore}}} \right)^3 \cos^3 \theta - (2 - 3 \sin \theta + \sin^3 \theta)} \right]^{1/3}$$


Critical Pressure of Permeation: Pressure required to permeate the dispersed fluid.

- \( R_{\text{pore}} \) increases \( \Rightarrow \) \( P_{\text{crit}} \) decreases
- \( R_{\text{drop}} \) increases \( \Rightarrow \) \( P_{\text{crit}} \) increases

Critical Pressure (bar)

Pore Diameter (µm)

\( \theta = 155^\circ \)

- Critical pressure as a function of drop size and pore size.

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Crossflowing Systems: Confined vs. Unconfined

Confined (T-junction)

<table>
<thead>
<tr>
<th>Channel driving force</th>
<th>Confined</th>
<th>Unconfined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td></td>
<td>Shear stress</td>
</tr>
<tr>
<td>Deformation and breakup</td>
<td>Due to pressure gradient</td>
<td>Due to shear stress and pressure.</td>
</tr>
<tr>
<td>Dispersion</td>
<td>Plugs</td>
<td>Droplets</td>
</tr>
<tr>
<td>Continuous phase flow rate</td>
<td>Very Low</td>
<td>high</td>
</tr>
<tr>
<td>Geometry</td>
<td>Strongly dependent on channel size</td>
<td>Roughly independent of channel size.</td>
</tr>
<tr>
<td>Droplets size</td>
<td>Easily controllable (mono-disperse)</td>
<td>Hardly controllable (poly-disperse)</td>
</tr>
</tbody>
</table>
Numerical Method and Important Parameters

CFD: Inexpensive, repeatable, instructive.
Flow solver: FLUENT
3D interface tracking: Volume of Fluid
Interface: outlined by cells containing both phases.
Surface Tension: “Continuum Surface Force”

<table>
<thead>
<tr>
<th>Important Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity Ratio (oil to water)</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Surface Tension Coefficient</td>
<td>( \sigma )</td>
</tr>
<tr>
<td>Contact angle (measured in oil)</td>
<td>( \theta )</td>
</tr>
<tr>
<td>Shear Rate</td>
<td>( \dot{\gamma} )</td>
</tr>
<tr>
<td>Drop to pore size ratio</td>
<td>( r_d / r_p )</td>
</tr>
</tbody>
</table>

Governing Equations

**Continuity**

\[
\frac{1}{\rho_q} \frac{\partial}{\partial t} \left( \alpha_q \rho_q \right) + \nabla \cdot \left( \alpha_q \rho_q \mathbf{v}_q \right) = S_{\alpha_q} + \sum_{p=1}^{n} \left( \dot{m}_{pq} - \dot{m}_{qp} \right)
\]

**Compatibility**

\[
\rho = \alpha_2 \rho_2 + (1 - \alpha_2) \rho_1
\]

**Momentum**

\[
\frac{\partial}{\partial t} \left( \rho \mathbf{v} \right) + \nabla \cdot \left( \rho \mathbf{v} \mathbf{v} \right) = -\nabla p + \nabla \cdot \left[ \mu \left( \nabla \mathbf{v} + \nabla \mathbf{v}^T \right) \right] + \rho \mathbf{g} + \mathbf{F}
\]
Validation of the Numerical Simulation

\[ \rho_1 = \rho_2 = 1, \quad \mu_1 = \mu_2 = 1, \quad r_d = 0.25, \quad \text{Shear} = 1, \quad \text{Re} = 0.0625 \]

<table>
<thead>
<tr>
<th>Ca</th>
<th>Present Solver</th>
<th>Li et al.†</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>0.2</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td>0.3</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>0.4</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
</tr>
</tbody>
</table>

\[ D = \frac{L-B}{L+B} \]

\[ Ca = \frac{r_d \dot{\gamma} \mu}{\sigma} \]

Results

Part 1:

1. Oil film permeation into pores of arbitrary cross-section (effect of transmembrane pressure).

2. Oil drop entry dynamics into circular pores under shear flow.
Critial Pressure of Permeation of Liquid Films inside Pores of Arbitrary Cross-Section

- Young-Laplace (pressure vs surface tension):
  \[ \Delta P = 2\sigma \kappa \]

- Mean curvature (no gravity) for arbitrary surface \( z \):
  \[
  2\kappa = \nabla \cdot \left( \frac{\nabla z}{\sqrt{1 + |\nabla z|^2}} \right)
  \]

- Boundary condition (imposed contact angle):
  \[
  \cos \theta = \mathbf{n} \cdot \left( \frac{\nabla z}{\sqrt{1 + |\nabla z|^2}} \right)
  \]

Integrating over cross-section:

\[
2\kappa = \frac{C_p \cos \theta}{A_p}
\]

Young-Laplace

Critical Pressure \( P_{\text{crit}} \):

\[
P_{\text{crit}} = \frac{\sigma C_p \cos \theta}{A_p}
\]

\( C_p \): pore circumference
\( A_p \): pore CS area
Numerical Simulation of Thin Oil Film on a Rectangular Pore

Lower aspect ratio: higher critical pressure.

Excellent agreement with critical pressure formula.

For square cross-section interface is spherical.

For 90 degrees: zero curvature + zero pressure gradient.

For 150 degrees no steady shape. Corners pinned.

\[ \frac{2\kappa}{P_{\text{crit}}} = \nabla \cdot \left( \frac{\nabla z}{\sqrt{1 + |\nabla z|^2}} \right) \]

\[ P_{\text{crit}} = 2\sigma \cos \theta \left( \frac{1}{w} + \frac{1}{l} \right) \]

\[ \lambda = 2.45, \ \sigma = 19.1 \text{ mN/m}, \ \theta = 120^\circ \]
Numerical Simulation of Thin Oil Film on an Elliptical Pore

- Interface inside an elliptical pore is not spherical.
- Excellent agreement with critical pressure formula.
- Infinitely long ellipse: higher critical pressure than infinitely long rectangle.
- If aspect ratio > 1.635 there is a critical contact angle above which the interface cannot remain attached.

\[ h = \frac{(a - b)^2}{(a + b)^2} \]

Using Ramanujan’s formula for ellipse perimeter (0.04% error):

\[ P_{\text{crit}} \approx \frac{(a + b)}{ab} \left[ 1 + \frac{3h}{10 + \sqrt{4 - 3h}} \right] \sigma \cos \theta \]

\[ \lambda = 2.45, \sigma = 19.1 \text{ mN/m}, \theta = 120^\circ \]
Part 1:

1. Oil film permeation into pores of arbitrary cross-section.

2. Oil drop entry dynamics into circular pores under shear flow (effect of shear rate and transmembrane pressure).
Numerical Simulation of Oil Droplet on a Circular Pore with No Crossflow

- Excellent agreement with critical pressure formula.
- Critical pressure increases with drop size and decreases with pore size.
- Drop entry dynamics inside pore slows down significantly when approaching critical pressure.
- Critical pressure of an infinitely large drop corresponds to an infinite oil film.

Note: If contact angle is 90 degrees, the predicted critical pressure is negative, meaning that the drop will penetrate the pore in the absence of transmembrane pressure.

$$P_{\text{crit}} = 2\gamma \frac{\cos(\theta)}{r_{\text{pore}}} \left(1 - \left(\frac{4 \left(\frac{r_{\text{drop}}}{r_{\text{pore}}}\right)^3}{2 + 3 \cos \theta - \cos^3 \theta} \right)^{\frac{1}{3}}\right)$$

Where:
- \(\lambda = 2.45\)
- \(\sigma = 19.1 \text{ mN/m}\)
- \(\theta = 135^\circ\)


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Computational setup for sheared droplet on membrane surface with a circular pore

- Channel dimensions chosen to minimize finite size effects.
- Moving top wall induces linear shear flow.
- Pressure outlet at the bottom of pore controls transmembrane pressure.
- Periodic boundary condition to ensure accurate shear flow.

\[
Ca = \frac{\mu \dot{\gamma} r_d}{\sigma} \leq 0.03 \quad \text{Re} = \frac{\rho_w \dot{\gamma} r_d^2}{\mu_w} \leq 0.5
\]

The mesh consists of hexagonal grids and contains 30 cells along pore diameter.
With increasing shear rate, critical permeation pressure increases.

Breakup is roughly independent of the transmembrane pressure.

The yellow line denotes the critical pressure of permeation (Focus of Part 2).

Volume of leakage: linearly dependent on transmembrane pressure.

Flow inside pore follows Hagen-Poiseuille.

For 100% rejection rate breakup should be avoided.

\[ \Delta P = \frac{8 \mu LQ}{\pi r^4} \]

\[ U_{\text{interface}} \propto \Delta P \]

\[ r_{\text{pore}} = 0.2 \, \mu m, \quad r_{\text{drop}} = 0.9 \, \mu m, \quad \lambda = 2.45, \quad \sigma = 19.1 \, mN/m, \quad \theta = 135^\circ, \quad \text{Shear} = 5 \times 10^5 \, s^{-1} \]
Results

Part 2:

1. Problem statement.
2. Effect of confinement on drop dynamics.
3. Effect of viscosity ratio on drop dynamics.
4. Effect of surface tension coefficient on drop dynamics.
5. Effect of contact angle on drop dynamics.
6. Effect of drop size on drop dynamics.
**Problem Statement: Interaction of an Oil Droplet with a Membrane Pore**

- **Young-Laplace Equation**: pressure drop across an interface:

  \[ \Delta P = 2k\sigma \]

- Motion of interface inside the pore is slow.

- The interface inside the channel is highly dynamic.

\[ P_1 - P_3 = P_1 - P_2 + P_2 - P_3 \]
Analytical formulation: Critical Capillary Number and Critical Pressure

\[ D \approx F_{\sigma} \]

\[ D \propto f_D(\lambda) \mu \dot{\gamma} r_d^2 \]
\[ F_{\sigma} \propto \sigma r_p \]

\[ f_D(\lambda) = \frac{2 + 4.51\lambda}{1 + 1.05\lambda} \]

For a hemispherical drop on solid surface\(^\dagger\)

\[ T \propto (P_{cr} - P_{cr0})A_p r_d \]

\[ T = f_T(\lambda)\pi \mu \dot{\gamma} r_d^3 \]

\[ f_T(\lambda) = \frac{2.19\lambda}{1 + 0.90\lambda} \]

\[ Ca_{cr} \propto \frac{1}{f_D(\lambda)\bar{r}} \]
\[ \bar{r} = r_d / r_p \]

\[ P_{cr} - P_{cr0} \propto \frac{f_T(\lambda)\sigma \bar{r} Ca}{r_p} \]

Droplet already deposited on the pore.

Bottom of pore is blocked.

Symmetry boundary condition solves half of the domain. As a result saves time.

Moving top wall generates linear shear flow.

A hybrid mesh with coarse tetrahedral meshes away from the pore and fine hexagonal meshes near the pore is used.

Grid independency check is confirmed

Basic parameters: $r_{\text{pore}} = 0.5 \mu m$, $r_{\text{drop}} = 2.0 \mu m$, $\lambda = 1$, $\sigma = 19.1$ mN/m, $\theta = 135^\circ$
Results

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Effect of Confinement on Drop Dynamics Near Circular Pores

- High channel height: high computational cost.
- Low channel height: not purely unconfined.
- Optimum channel height?
- Highly confined droplet elongates more.
- Higher confinement -> lower breakup capillary number.

- Optimum confinement ratio \( \sim 0.428 \)

\[
\gamma = 1.5 \times 10^5 \text{ s}^{-1}
\]

\[
C_{cr} = 0.032 - 0.023
\]

\[
\text{Confinement Ratio } (H_d/H_{ch})
\]

- \( r_{pore} = 0.5 \text{ µm} \), \( r_{drop} = 2.0 \text{ µm} \), \( \lambda = 1 \), \( \sigma = 19.1 \text{ mN/m} \), \( \theta = 135^\circ \)
Results

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5. Effect of contact angle on drop dynamics.
6. Effect of drop size on drop dynamics.
Effect of Viscosity Ratio on Breakup Time of the Droplet

- Deformation time scale: measure of drop deformation time.
- Solid body -> infinite deformation time scale.
- Computed breakup time increases with viscosity ratio.
- Drop profiles at breakup match very closely (self-similarity).

$r_{\text{pore}} = 0.5 \, \mu m$, $r_{\text{drop}} = 2.0 \, \mu m$, $\sigma = 19.1 \, mN/m$, $\theta = 135^\circ$
Effect of Viscosity Ratio on Drop Dynamics Near Circular Pores

With increasing capillary number $Ca$, critical permeation pressure increases.

- High viscosity ratio = high critical pressure.
- Higher viscosity ratio = easier breakup.
- Droplets with low viscosity ratio should be avoided for emulsification.
- Increasing $Ca$ increases deformation.

$r_{pore} = 0.5 \, \mu m$, $r_{drop} = 2.0 \, \mu m$, $\sigma = 19.1 \, mN/m$, $\theta = 135^\circ$

$\lambda = \frac{\mu_{oil}}{\mu_{water}}$

$Ca = \frac{\mu yr_d}{\sigma}$

$% \text{Inc} P_{crit} = \left( P_{crit} - P_{crit_0} \right) / P_{crit_0} \times 100$
Self-similarity of the Drop Behavior for any Viscosity Ratio

- Drops with different viscosity ratio behave similarly.
- Capillary number is multiplied by $f_D(\lambda)$ because highly viscous drops break at lower shear rates.
- Critical pressure is divided by $f_T(\lambda)$ because highly viscous drops have higher critical pressure.

\[ Ca_{cr} \times f_D(\lambda) \propto \frac{1}{\bar{r}} \]

\[ \frac{(P_{cr} - P_{cr_0})}{f_T(\lambda)} \propto \frac{\sigma \bar{r} Ca}{r_p} \]

\[ f_D(\lambda) = \frac{2 + 4.5 \lambda}{1 + 1.05 \lambda} \]

\[ f_T(\lambda) = \frac{2.19 \lambda}{1 + 0.90 \lambda} \]

$r_{pore} = 0.5 \mu m$, $r_{drop} = 2.0 \mu m$, $\sigma = 19.1 \text{ mN/m}$, $\theta = 135^\circ$
Part 2:

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5. Effect of contact angle on drop dynamics.
6. Effect of drop size on drop dynamics.
Breakup time of droplet for different surface tension coefficients

- Deformation time scale decreases with surface tension.
- Breakup time linearly increases with deformation time scale.
- Drops with higher surface tension breakup faster.
- Drop profiles at breakup match exactly (self-similarity).

Deformation to steady state

Quasi steady breakup

Surface Tension

- $\sigma = 38.2$ mN/m
- $\sigma = 28.6$ mN/m
- $\sigma = 19.1$ mN/m
- $\sigma = 14.3$ mN/m
- $\sigma = 9.55$ mN/m

$r_{pore} = 0.5$ µm, $r_{drop} = 2.0$ µm, $\lambda = 1$, $\theta = 135°$
Effect of Surface Tension on Drop Dynamics Near Circular Pores

- Surface tension resists external forces (pressure, shear stress, etc.).
- Increasing surface tension coefficient increases critical pressure of permeation.
- Drops with high surface tension break at higher shear rates.
- Microfiltration -> high surface tension.

\[ r_{\text{pore}} = 0.5 \, \mu \text{m}, \quad r_{\text{drop}} = 2.0 \, \mu \text{m}, \quad \lambda = 1, \quad \theta = 135^\circ \]

\[ P_{\text{crit}} = 2\gamma \frac{\cos(\theta)}{r_{\text{pore}}} \left[ 1 - \left( \frac{2 + 3 \cos \theta - \cos^3 \theta}{4 \left( \frac{r_{\text{drop}}}{r_{\text{pore}}} \right)^3 \cos^3 \theta - (2 - 3 \sin \theta + \sin^3 \theta)} \right)^{1/3} \right] \]

Nazzal & Wiesner, (1996)
Self-similarity of the Drop Behavior for any Surface Tension

- Dividing \( Ca \) and \( P_{cr} \) by the surface tension coefficient makes results independent from surface tension.

- Breakup at \( Ca \approx 0.032 \).

\[
% \text{Inc} P_{crit} = \frac{(P_{crit} - P_{crit_0})}{P_{crit_0}} \times 100
\]

\[
P_{crit_0} \propto \sigma
\]

\[
Ca = \frac{\mu \dot{\gamma} r_d}{\sigma}
\]

\( r_{pore} = 0.5 \, \mu m, \quad r_{drop} = 2.0 \, \mu m, \quad \lambda = 1, \quad \theta = 135^\circ \)
Results

Part 2:

1. Problem statement.
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4. Effect of surface tension coefficient on drop dynamics.
5. Effect of contact angle on drop dynamics.
6. Effect of drop size on drop dynamics.
Effect of Contact Angle on Drop Dynamics Near Circular Pores

- Increasing contact angle increases the critical pressure of permeation.
- Breakup shear rate is roughly independent of contact angle.
- Contact angles of 115 degrees results in a 21% increase in critical pressure before breakup while 155 increases only 6%.

$r_{pore} = 0.5 \, \mu m, \ r_{drop} = 2.0 \, \mu m, \ \lambda = 1, \ \sigma = 19.1 \, mN/m$

$P_{crit} = 2\gamma \left(\frac{\cos(\theta)}{r_{pore}}\right) \left\{1 - \left\{\frac{2 + 3\cos \theta - \cos^3 \theta}{4 \left(\frac{r_{drop}}{r_{pore}}\right)^3 \cos^3 \theta - (2 - 3\sin \theta + \sin^3 \theta)}\right\}^{\frac{1}{3}}\right\}$

Nazzal & Wiesner, (1996)
Pressure Increase vs. Capillary Number for Different Contact Angles

- Increase in critical pressure roughly independent of contact angle.
- Lower contact angle drops are able to elongate more.

\[
P_{cr} - P_{cr0} \propto \frac{\sigma \gamma \mu r_d}{Ca}
\]

Contact Angle

WATER

OIL

SOLID SURFACE

\(Ca = \frac{\mu \gamma r_d}{\sigma}\)

\(T = f_T(\lambda) \pi \mu \gamma r_d^3\)

\(r_{pore} = 0.5 \mu m, \ r_{drop} = 2.0 \mu m, \ \lambda = 1, \ \sigma = 19.1 \text{ mN/m}\)
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6. Effect of drop size on drop dynamics.
Effect of Drop size on Entry Dynamics Near Circular Pores

- Larger drops: higher critical pressure.
- Larger drops: lower breakup shear rate.
- Increase in critical pressure before breakup is higher for smaller droplets.
- Increase in critical pressure is higher for larger drops for the same shear rate.

Nazzal & Wiesner, (1996)

\[ P_{crit} = 2\gamma \frac{\cos(\theta)}{r_{pore}} \times [1 - \left\{ \frac{2 + 3\cos(\theta) - \cos^3(\theta)}{4\left(\frac{r_{drop}}{r_{pore}}\right)^3\cos^3(\theta) - (2 - 3\sin(\theta) + \sin^3(\theta))} \right\}^{1/3}] \]

- \( r_{pore} = 0.5 \mu m, \ \lambda = 1, \ \sigma = 19.1 \text{ mN/m}, \ \theta = 135^\circ \)
Self-similarity of the Drop Behavior for any Drop Size

- Modified breakup capillary number is independent of drop size.
- Smaller drops deform more near the pore.
- New non-dimensional number identified.

\[
P_{cr} - P_{cr_0} \propto \frac{f_T(\lambda)\sigma \bar{r} Ca}{r_p}
\]

\[
Ca_{cr} \times \bar{r} \propto f_D(\lambda)
\]

\[
f_D(\lambda) = \frac{2 + 4.5 \lambda}{1 + 1.05 \lambda}
\]

\[
f_T(\lambda) = \frac{2.19 \lambda}{1 + 0.90 \lambda}
\]

\[r_{pore} = 0.5 \, \mu m, \, \lambda = 1, \, \sigma = 19.1 \, mN/m, \, \theta = 135^\circ\]

\[
\bar{r} = \frac{r_d}{r_p}
\]
Summary

Increasing the parameters in green boxes improve rejection of oil

Increasing the parameters in green boxes increases chance of breakup

\[ \lambda = \frac{\mu_{\text{oil}}}{\mu_{\text{water}}} \]

\[ P_{cr} - P_{cr_0} \propto \frac{f_T(\lambda)\sigma\bar{r}Ca}{r_p} \]

\[ Ca_{cr} \propto \frac{1}{f_D(\lambda)\bar{r}} \]

\[ \bar{r} = \frac{r_d}{r_p} \]

Important Conclusions

✓ A formula for calculation of critical pressure of entry of an oil film was derived and validated numerically.

✓ Increasing shear flow, increases critical pressure of permeation.

✓ Depending on transmembrane pressure and shear rate, a droplet near the pore of a membrane will be washed away, break, or permeate the pore.

✓ Confined drops break at lower shear rates compared to unconfined drops.

✓ Critical pressure for crossflow microfiltration increases with viscosity ratio, surface tension coefficient, and drop size.

✓ Increasing shear rate, viscosity ratio, and size of the drop increases chance of breakup.

✓ Increasing surface tension coefficient decreases chance of breakup.

✓ New dimensionless variables were introduced that result in solutions independent of various parameters.
Proposed Future Work

Numerical simulation of a droplet on a membrane surface with hydrophobicity gradient.

- Constant contact angle ($\theta = 137^\circ$).
- Droplet pulled towards the pore due to the induced flow.

- Linear contact angle gradient along the membrane surface.
  - $\theta = 150^\circ$ at pore entrance.
  - $\theta = 70^\circ$ at far right side.
  - Droplet driven away from the pore despite the induced flow towards the pore.

$r_{\text{pore}} = 0.2 \, \mu m$, $r_{\text{drop}} = 0.9 \, \mu m$, $\lambda = 2.45$, $\sigma = 19.1 \, \text{mN/m}$, $\Delta P = -70 \, \text{kPa}$
Proposed Future Work (continued)

- Droplet interaction with multiple pores.
- Lateral pressure driven flow.
- Droplet released upstream.

\[ r_{\text{pore}} = 5 \, \mu\text{m}, \quad r_{\text{drop}} = 33 \, \mu\text{m}, \quad \lambda = 1, \quad \sigma = 19.1 \, \text{mN/m}, \quad \theta = 180^\circ, \quad \Delta P_{\text{pore}} = -10 \, \text{kPa}, \quad \Delta P_{\text{lat}} = 1 \, \text{kPa} \]

- Multiple droplets interacting with pores of different size.
- Constant transmembrane pressure.
- Higher flow rate through larger pore.
- Drops migrate towards larger pore.

\[ r_{\text{pore\_large}} = 0.1 \, \mu\text{m}, \quad r_{\text{pore\_small}} = 0.05 \, \mu\text{m}, \quad r_{\text{drop}} = 0.4 \, \mu\text{m}, \quad \lambda = 2.45, \quad \sigma = 19.1 \, \text{mN/m}, \quad \theta = 180^\circ, \quad \Delta P_{\text{pore}} = -150 \, \text{kPa} \]
Droplet on slotted pore.

- Hypothesis: critical pressure increases by increasing inclination angle.

\[ a = 1 \, \mu m, \quad b = \infty, \quad r_{\text{drop}} = 2 \, \mu m, \quad \lambda = 1, \quad \sigma = 19.1 \, \text{mN/m}, \quad \theta = 135^\circ, \quad \Delta P_{\text{pore}} = -38 \, \text{kPa} \]

- Droplet on inclined pore.
- Hypothesis: critical pressure increases by increasing inclination angle.

\[ r_{\text{pore}} = 0.5 \, \mu m, \quad r_{\text{drop}} = 2 \, \mu m, \quad \theta_{\text{pore}} = 30^\circ, \quad \lambda = 1, \quad \sigma = 19.1 \, \text{mN/m}, \quad \theta = 135^\circ, \quad \text{Shear} = 5 \times 10^5 \, \text{s}^{-1} \]
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