# Homework 2: Vector calculus

1. R.T.P. that
   (i) $\nabla \cdot (\mathbf{r}f(r)) = 3f(r) + r\frac{\mathrm{d}f}{\mathrm{d}r}$.
   (ii) $\nabla \cdot (\mathbf{r}r^{n-1}) = (n+2)r^{n-1}$.
   (iii) $\mathbf{A} \times (\nabla \times \mathbf{B}) = \nabla_B(\mathbf{A} \cdot \mathbf{B}) - (\mathbf{A} \cdot \nabla)\mathbf{B}$.
   (iv) $\nabla \times (f\mathbf{V}) = f\nabla \times \mathbf{V} + \nabla f \times \mathbf{V}$.
   (v) $\nabla \times (\mathbf{r}f(r)) = 0$.

2. (a) Show that
   $$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t},$$
   where $\mathbf{E}$ is the electric field and $\mathbf{A}$ the magnetic vector potential, is irrotational and, therefore,
   $$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}.$$

   (b) Show that the Lorentz force on a point charge
   $$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$
   can be rewritten as
   $$\mathbf{F} = q\left[-\nabla\phi - \frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} + \nabla(\mathbf{A} \cdot \mathbf{v})\right].$$

3. Write $\nabla\psi$, $\nabla \cdot \mathbf{V}$, and $\nabla^2\mathbf{V}$ in cylindrical polar coordinates.