

## Homework 10: Sturm-Liouville Theory

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1. (a) Write down the Laguerre equation. [2]
- (b) Give a physics problem where it arises. [2]
- (c) Rewrite it in self-adjoint form. [4]
- (d) Give the  $p$ ,  $q$ ,  $\lambda$  and  $w$  parameters of the Sturm-Liouville equation. [2]

## Solution

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1. (a)

$$xy'' + (1-x)y' + \alpha y = 0.$$

(b) In the study of the hydrogen atom.

(c) The integrating factor is

$$R = \frac{1}{x} e^{\int dx(1-x)/x} = e^{-x}.$$

(d)

$$\frac{d}{dx} \left( x e^{-x} \frac{dy(x)}{dx} \right) + \alpha e^{-x} y(x) = 0.$$

Thus

$$p(x) = x e^{-x}, q(x) = 0, \lambda = \alpha, \rho(x) = e^{-x}.$$