Homework 1: Real series

1. Establish the convergence or not of the following series:
   (i) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$; (ii) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$; (iii) $\sum_{n=1}^{\infty} \frac{1}{2n(2n-1)}$; (iv) $\sum_{n=1}^{\infty} \frac{1}{[n(n+1)]^{1/2}}$; (v) $\sum_{n=0}^{\infty} \frac{1}{(2n+1)}$.

Homework Solution 1

1. divergent: compare with $1/n$.
$$\frac{1}{\ln n} > \frac{1}{n}.$$
2. divergent: ratio.
$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{n+1}{10} \to n.$$
3. convergent: Raabe.
$$R_n = n\left(\frac{u_n}{u_{n+1}} - 1\right) = n\left(\frac{2(n+1)(2n+3)}{2n(2n+1)} - 1\right) = n\frac{8n+3}{2n(2n+1)} \to 2.$$
Also,
$$\frac{1}{2n(2n-1)} = -\frac{1}{2n} + \frac{1}{2n-1}.$$
Then use integral test or rewrite sum as an alternating series.
4. divergent: Gauss
$$\frac{u_n}{u_{n+1}} = 1 + \frac{1}{n}.$$
5. divergent: Gauss
$$\frac{u_n}{u_{n+1}} = 1 + \frac{1}{n}.$$