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Spatial and temporal frequency in animated mimic displays

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Animated mimic displays provide analogical representations of the flow of information or resources between system components. For example, the transfer of fuel from one tank to another in an airplane could be represented by fluid running through the connecting pipe, rather than an analog meter or digital display. The quality of apparent motion is critical: does the subjective impression of flow that results from viewing the display correspond to the physical rate of flow that exists in the underlying domain? Previous research has revealed potential problems with display configurations modeled on prominent examples in the literature, including ambiguity with respect to both the direction and the rate of flow. Two psychophysical experiments were conducted to 1) investigate these potential problems more thoroughly and 2) examine two critical design parameters. Observers performed a rate-matching task with various combinations of spatial frequency, temporal frequency, and luminance contrast. The findings were atypical: the usual "bandpass" pattern (better performance at intermediate frequencies, relative to high or low frequencies) was obtained only for spatial frequency, and only when luminance contrast was low. In addition, performance was more closely tied to specific combinations of spatial and temporal frequency than to velocity. An interpretation based on Fourier's theorem is discussed and alternative display designs are proposed. Finally, design guidelines for spatial and temporal frequency are provided.

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Running Title: ANIMATED MIMICS

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INTRODUCTION

One critical issue in system design is how to exploit the recent advances in computer technology to maximize overall human-machine system performance. Many forms of decision support have been developed, including automation, expert systems, and calculated support (e.g., predictor variables). By and large these forms of decision support can be considered “prosthetic” in nature, in that the goal is to replace the human problem solver (see [1] for a detailed discussion of the consequences of this design approach for expert systems). A complementary design approach for decision support is to provide interface resources that enhance, rather than replace, the human problem solver’s capabilities.

One form of decision support that is consistent with this approach has been referred to as “representation aiding” [2,3,4]. Representation aiding supports an individual by providing computerized graphic representations of a domain, thereby taking advantage of the efficient pattern-recognition capabilities and flexible decision-making processes of the human. This form of decision support has the potential to increase the system’s capability to respond to unforeseen, but inevitable, circumstances [5].

The successful design of representation aids is dependent upon an understanding of the complex relationships between 1) the specific characteristics of the underlying domain, 2) the specific characteristics of the representations of the domain (e.g., displays) that are provided, and 3) the specific perceptual and cognitive characteristics of problem solvers [1,6,7,8,9]. One perspective that problem solvers in complex, dynamic domains must consider is related to the physical components that comprise the system and the flow of information or resources between these components. Rasmussen [7] has referred to this perspective as the level of “physical function” in his abstraction hierarchy.

The displays that have been developed to present physical function are more commonly referred to as “mimic” or “pictoral” displays [10,11,12,13]. When the flow of information or resources between system components is represented analogically these displays may be referred to as “animated mimics.” A representative example is illustrated in Fig. 1. The display contains representations of the system components (reactor core, steam generator, and steam turbine), the pipes that connect them, and the flow of resources between them (e.g., steam flow, from steam generator to steam turbine).

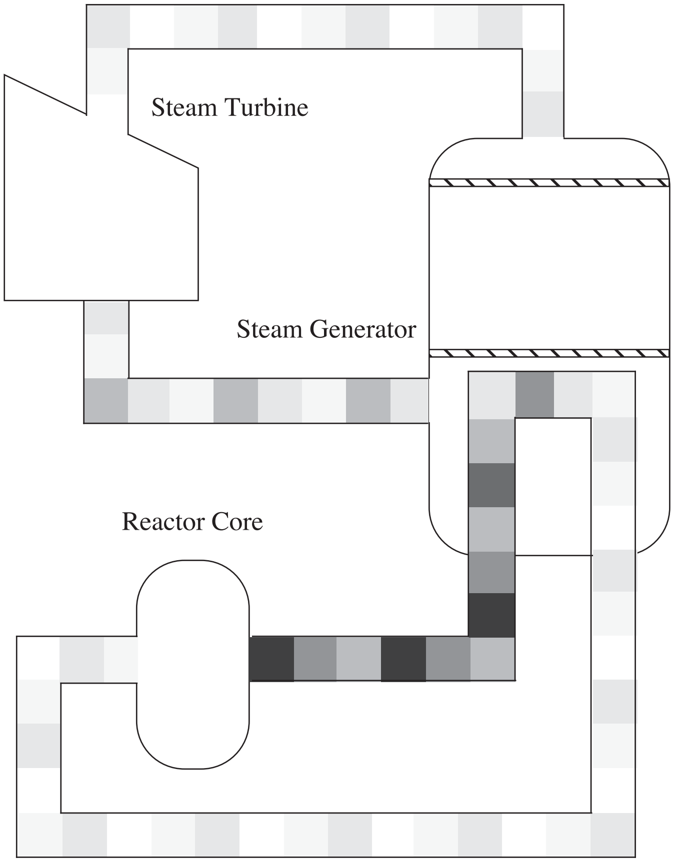

Figure 1. A representative animated mimic display with a stairstep luminance profile.

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Animated functional mimic displays have the potential to improve the quality of decision support in complex, dynamic domains. One benefit is the potential to improve the efficiency of training, as illustrated in STEAMER (a computer-based instructional system for propulsion engineering, [10,11,12]). The work of Hollan and his colleagues emphasize that graphical displays (including animated functional mimics) can facilitate appropriate mental models and effective causal reasoning skills by providing "dynamic graphical explanations" [11, p. 19]. There are also potential benefits for real-time performance. Animated mimic displays may facilitate 1) the identification of system state, 2) the identification of alternative resources required to recover from trouble, and 3) feedback regarding the effectiveness of control input. See Bennett [2] for a more detailed description of the potential benefits.

A critical issue in the design of animated mimic displays is the quality of the animation that is produced: is there an effective mapping between the flow rate that exists in the actual domain and the subjective impression of motion that is perceived by an observer viewing the display? The results of an initial study [2] indicated potential problems with a display configuration modeled on the STEAMER displays ([10,11,12], e.g., Fig. 1). The subjects' task was to match the rate of motion (varied across trials) in a standard display by changing the rate of motion in a comparison display. Observers reported that the apparent motion in these displays could be seen in either direction through conscious shifts in attention. This could account for the rare (1-2% of trials), but large errors in accuracy that were observed (observers may have focused on the inappropriate direction of motion). Thus, an extreme case of an inappropriate mapping between the representation and the domain could occur in an applied setting with displays designed along these lines (e.g., an operator may actually be draining a tank, when he/she believes that it is being filled).

Before considering a theoretical explanation of this bi-directional apparent motion a few general definitions will be provided. Spatial frequency refers to the number of repetitions of a wave form, or cycles, that fall within one degree of visual angle (expressed in cycles/degree). Temporal frequency refers to the number of cycles that pass a fixed point during a one second interval (measured in cycles/second, or Hz). The term "fundamental" is perhaps a more appropriate term to specify both spatial and temporal frequency with more complex, non-sinusoidal patterns like those illustrated in Fig. 1 (e.g., fundamental spatial frequency, see the following paragraph). However, the shorter terms will be used for simplicity. Velocity is the temporal frequency divided by the spatial frequency, and refers to the number of degrees (visual angle) that a wave form moves during a one second interval (units are degrees/second). The amplitude of a wave form refers to the difference in luminance between the peak and trough of a wave form; a typical unit is candelas per meter squared (cd/m^2).

A potential explanation of why the display configuration depicted in Fig. 1 may produce ambiguous apparent motion is based on insights from Fourier's theorem. The upper two panels of Figure 2 illustrate a sinusoidal wave form (left) and a stairstep wave form. Fourier's theorem demonstrates that a wave form of any shape and amplitude can be synthesized by combining an infinite number of sinusoidal wave forms that have particular spatial frequencies and amplitudes [14,15]. A Fourier analysis of the stairstep wave form was completed (see Fig. 2 and Table 1). This wave form can be synthesized by combining the sine waves of all harmonics of the stairstep wave, except for those harmonics that are multiples of three (e.g., F, $2F$, $4F$, $5F$, $7F$, $8F$, $10F$, $11F$, ... etc., see Table 1a). The amplitude of each successive sine wave is a fraction of the stairstep wave's amplitude (e.g., $3A / \pi$, $3/2A / \pi$, $3/4A / \pi$, $3/5A / \pi$, $3/7A / \pi$, $3/8A / \pi$, $3/10A / \pi$, $3/11A / \pi$, ..., etc., see Table 1b). The lower part of Fig. 2 illustrates the results of the Fourier analysis graphically. A partial synthesis of the stairstep wave form is shown: the panels on the left illustrate successive sinusoidal harmonics (F, $2F$, and $11F$ are present); the panels on the right illustrate the changes in the synthesized wave form as each successive harmonic is added. By the 11th harmonic the stairstep wave form is approximated, and adding additional harmonics would provide an even closer approximation.

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Insert Table 1 about here

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This would be only an interesting mathematical phenomenon, except that there is ample evi-

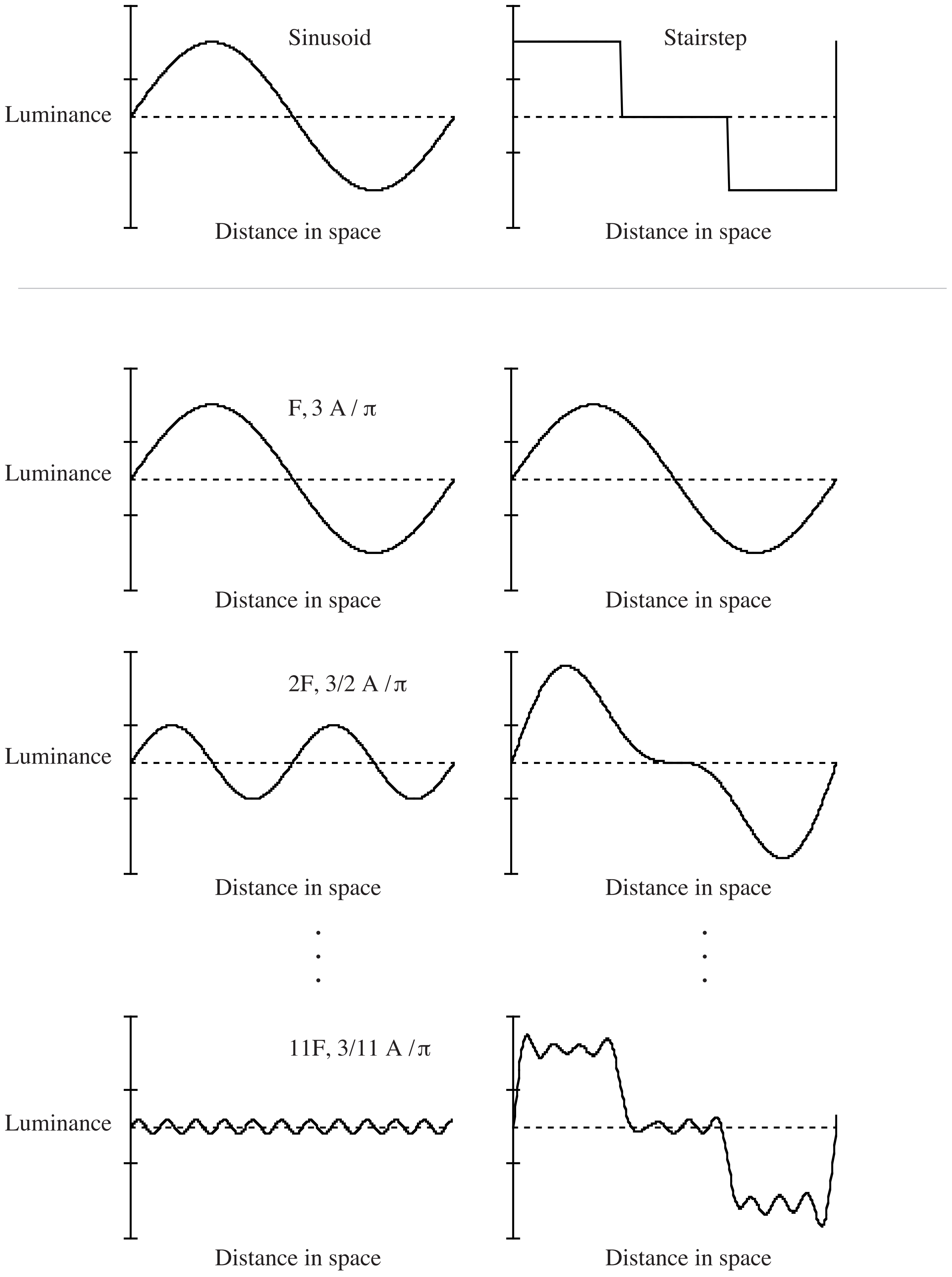

Figure 2. Sinusoid and stairstep wave forms; Fourier's theorem. The upper portion of the figure illustrates a sinusoid (left) and a stairstep (right) wave form. The lower portion of the figure illustrates the synthesis of a stairstep wave form from sinusoidal harmonics. The first, second, and eleventhth harmonic are illustrated in the panels on the left. The panels on the right illustrate the shape of the synthesized wave form as each harmonic is added.

dence that the early visual system performs a crude approximation to a Fourier analysis (patterns are analyzed on the basis of their underlying frequency content) [14,16,17]. Braddick, Campbell, and Atkinson [18] provide a thorough review of the physiological and behavioral research that supports this theoretical position. Adelson [27] performed an experiment that is particularly relevant to the present study; Anstis ([26], p. 16-12, emphasis added) summarizes and interprets the results:

> Adelson (1982) ... considered a grating with a square-wave luminance profile that jumped successively through one-quarter of a spatial period (half a bar width) to the right. This is equivalent to a set of harmonic gratings of relative frequencies 3, 5, 7... jumping through 3/4, 5/4, 7/4 ... spatial periods to the right. Note that the third harmonic's jump of 3/4 spatial period to the right is identical to a jump of 1/4 period to the left. <u>Higher harmonics will jump every which way</u>, but their amplitudes are too low to have much perceptual effect. Adelson then presented a square-wave grating minus its fundamental and made it jump successively through 1/4 cycle to the right. Observers saw it as apparently jumping to the left. This result would not be predicted if the visual system extracted edges or other pattern features, which do jump to the right. However, a system acting as a Fourier analyzer would give the result he obtained. It may sound intuitively unlikely that the visual system would break a jumping pattern down into its harmonics, figure out the phase shift of each harmonic, and put them together again to give a pure spatial (minimum phase) jump, but that is what his results suggest.

Thus, Adelson found that a square wave with the fundamental harmonic removed produced apparent motion in the direction opposite to the physical update. A similar analysis (which will be referred to as the frequency analysis hypothesis) provides a potential explanation for the bidirectional apparent motion that occurs with the stairstep wave form. Both the sinusoidal harmonics of the wave form (see Table 1) and the phase shift introduced by the animation technique must be considered. Color table animation techniques [19,20] are used to produce apparent motion in the animated mimic display: individual graphical elements (e.g., each square in Fig. 1) are associated with indexes in color table RAM, the RGB values in these indexes are changed, and the color table is reset. Thus, an update to the display always produces the same physical change: a 1/3 cycle phase shift in only one direction.

In terms of sinusoidal harmonics, however, a screen update will produce a 1/3 cycle phase shift only for the first sinusoidal harmonic (i.e., the fundamental). Consider the left panel of Fig.

3. The bold sinusoidal wave form represents the position of the fundamental harmonic before the update, the vertical dotted line represents the 1/3 phase shift caused by updating the stairstep wave form, and the lighter sinusoidal wave form represents the position of the harmonic after an update. Thus, for the first harmonic a screen update produces a physical displacement of 1/3 cycle from left to right, and the visual system interprets this as apparent motion in the same direction.

However, the implications of a screen update are quite different for all subsequent harmonics. Consider the second harmonic. Because this harmonic is twice the spatial frequency of the fundamental, a screen update produces a 2/3 cycle phase shift from left to right (illustrated in the right panel of Fig. 3). In this case the apparent motion that results is in the opposite direction. The visual system interprets this change as motion to the left (traversing a smaller distance), as opposed to motion to the right (traversing a larger distance). Note that the rate of motion (velocity) also changes, as illustrated by the shorter distance travelled by the harmonic in the right panel. Similarly, each additional pair of harmonics would produce apparent motion that alternates in direction and decreases in velocity (see Table 2). For example, the next pair of harmonics would produce left-to-right apparent motion ($4F$) and right-to-left apparent motion ($5F$). Thus, it is possible that observers could base their percept of motion on the higher sinusoidal harmonics, and that they could focus on these harmonics at their discretion. In particular, the fundamental and the first harmonic would have the highest amplitudes, and would be moving in opposite directions.

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Insert Fig. 3 about here

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Insert Table 2 about here

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The present experiments were conducted with dual research goals in mind. As the previous discussion indicates, the frequency analysis hypothesis provides one potential explanation of the problems associated with this particular type of animated mimic display. However, more solid experimental evidence should be obtained to support or refute this interpretation. It is also clear that spatial frequency, temporal frequency, and velocity are important design variables that are likely to have an impact on the effectiveness of animated functional mimics. Although the basic

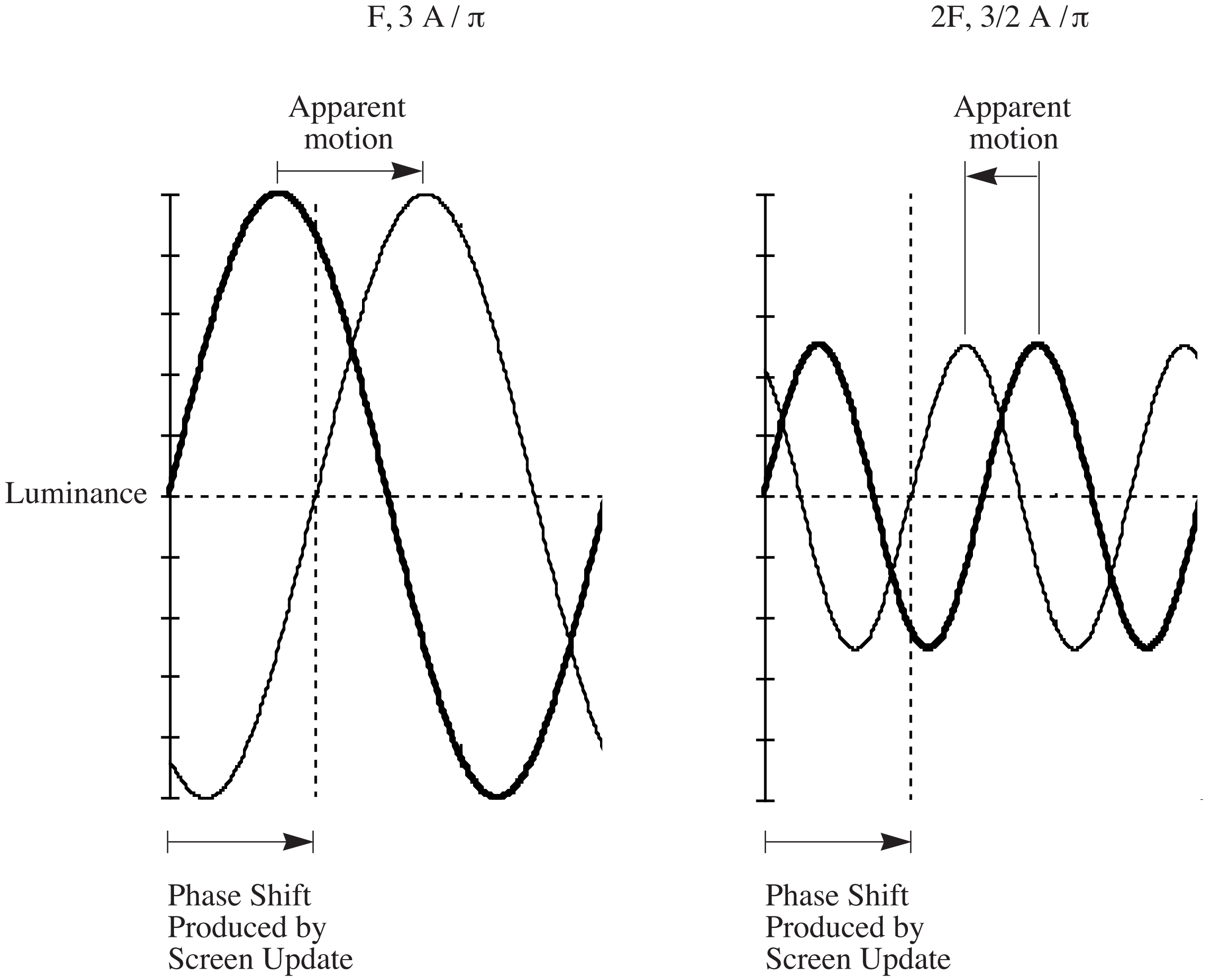

Figure 3. Bi-directional apparent motion in harmonics caused by screen update to stairstep wave form. The first harmonic (the fundamental frequency) is illustrated in the left panel. Updating the stairstep wave form produces a 1/3 phase shift from left to right, and produces apparent motion for this harmonic in the same direction. For the second harmonic an update produces a 2/3 phase shift. The visual system interprets this change as motion from right to left, and the rate of motion is half that of the fundamental.

vision literature has investigated these parameters fully (e.g., Kelly [22]), there are important differences that may preclude the direct translation of these results[1]. The first experiment investigated rate-matching performance as a function of changes in these three variables. The observers' task was to change the rate of apparent motion in a standard bar to match that of a comparison bar. Wave forms with stairstep luminance profiles were used. Eight levels of spatial and temporal frequency were combined factorially for a total of 64 trials per experimental session. The luminance contrast remained the same during an experimental session, but was lowered on each of six successive sessions.

EXPERIMENT 1

Method

Subjects. Nine observers (4 male and 5 female, all students at WSU) participated in the experiment and were paid $5.00 an hour. The observers ages ranged from 20 to 23 years of age and all observers had normal or normal-corrected vision. All nine observers had participated in previous experiments using similar procedures but different stimuli.

Apparatus. All experimental events were controlled by a general purpose laboratory computer (Sun Microsystem 4-110 Workstation). A 40.64 cm color video monitor (Sony Trinitron, model GDM-1604-15) was used to present the stimuli and experimental prompts. The resolution of the monitor was 1152 by 900 pixels, corresponding to 39.37 pixels/cm, and it had a refresh rate of 66 Hz, non-interlacing.

Stimuli. Two horizontal bars were presented on a medium grey background. Each bar was 19.51 cm wide and 0.38 cm high and separated vertically by a distance of 4.8 cm. The two bars were centered in the horizontal and vertical dimensions of the screen. A chin-rest was used to maintain observer viewing distance at 100 cm. Thus, each bar subtended a visual angle of 11.04 degrees horizontally and 13.07 arc min vertically, while the distance between the bars subtended a visual angle of 2.75 degrees.

Each bar contained graphical elements (in Fig. 1 these elements are square-shaped) that differed in luminance contrast (there were three levels of contrast that repeated every third graphical

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[1] See the discussion section for a more detailed discussion of these differences.

element, as in the horizontal connections in Fig. 1). This formed a stairstep wave form with abrupt changes in luminance. The luminance contrasts were shifted from the left to the right at discrete intervals, thus producing apparent motion in the bars. The lower bar was the “standard” bar, and its temporal frequency was an independent variable. The upper bar was the “comparison” bar and its temporal frequency was controlled by the observer.

There were eight levels of temporal frequency for the standard bar: 0.83, 1.00, 1.33, 2.00, 3.33, 6.00, 11.33, and 20 Hz. At the beginning of a trial the temporal frequency of the comparison bar was either 0.92, 1.67, 4.67, or 16.67 Hz (counterbalanced within temporal frequency of the standard bar). There were also eight levels of spatial frequency: 0.18, 0.36, 0.72, 1.43, 2.86, 5.73, 11.45, and 22.91 c/deg. One graphical element subtended 111.73, 55.88, 27.94, 13.97, 6.99, 3.49, 1.75, and 0.87 arc min degrees of horizontal visual angle at each respective level. The eight levels of spatial and temporal frequency were combined factorially for a total of 64 combinations. The resulting velocities, which ranged from 0.04 to 111.42 deg/sec, are listed in Table 3.

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Six separate sets of stimuli that differed in luminance contrast (from 1.13% to 32.45% contrast) were developed for each of the six experimental sessions. The chromaticity coordinates and luminance contrasts were measured separately for each of the three repeating graphical elements in a set. All measurements were made with a Minolta Chroma Meter (model CS101) which measured chromaticity in x and y coordinates (CIE) and luminance in cd/m^2. All chromaticity measurements were translated into the CIELUV u’ and v’ chromaticity coordinates using the formulas u’ = 4x / -2x +12y +3 and v’ = 9y / -2x +12 y + 3, as described in [21]. The Michelson formula was used to determine luminance contrast: C = (Lmax - Lmin) / 2 ($\overline{L}$), where Lmax is the maximum luminance, Lmin is the minimum luminance, and $\overline{L}$ is the mean luminance. The results of these measurements and analyses are listed in Table 4. It should be emphasized that the spatial frequency, and perceptual characteristics of the graphical elements in the standard and comparison bars were always equal during an experimental trial. The measurements for the background were u’ = 0.1994, v’ = 0.4662, and cd/m^2 = 4.01.

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<u>Procedure</u>. The experiment was conducted during a six-day period with one experimental session per day. The luminance contrast of the stairstep was systematically reduced across sessions by using stimuli sets 1 through 6 in successive order. During a previous experiment the observers were provided with both a written and a verbal explanation of the task, focusing on the details of the method of adjustment (see description below). The observers were instructed to respond as accurately and quickly as possible. They were informed that the stimuli would be slightly different in the present experiment, but that all other procedures would remain the same. The observers were seated in an enclosed room with flat-black walls, and during an experimental session all ambient lighting was removed. They were required to use a chin-rest to maintain a constant viewing distance.

The observer's task was to adjust the temporal frequency of the comparison bar to match that of the standard bar. Observers used an optical mouse to position the cursor over one of two labeled boxes ("start" or "stop") in the top left portion of the window, and clicked a mouse button to begin or end a trial. The temporal frequency of the comparison bar was increased or decreased by pointing and clicking two buttons containing upward or downward facing arrows. These buttons were located just above and to the left of the standard and comparison bars. The observer could not reverse the direction of motion during a trial or exceed the raw update of the monitor (66 Hz, corresponding to an upper limit of 22 Hz for the wave form).

A modified version of a standard psychophysical procedure (the method of adjustment) was used. The first input by an observer increased or decreased the temporal frequency of the comparison bar by a predetermined increment of 1.67 Hz. From that point the size of the increment depended upon both the current increment and the direction of previous observer input. An observer input in the opposite direction from the previous input (a reversal) changed the temporal frequency by half the current increment. An observer input in the same direction as the previous input changed the temporal frequency by the current increment, unless the two previous observer inputs were in the same direction. In this case the change in temporal frequency was double the current increment. At any point an observer could end a trial by clicking on the stop button.

Measures of both accuracy and latency (accurate to 1/100 of a second) were obtained for each experimental trial, and observers were provided with feedback for both measures. To summarize, in each of six experimental sessions an observer completed 64 trials: a factorial combination of the eight temporal frequencies (within-subjects factor) and eight spatial frequencies (within-subjects factor). The order of these trials was determined randomly in each experimental session. The luminance contrast was systematically reduced by using stimulus sets 1 through 6 in the corresponding experimental sessions (within-subjects factor).

RESULTS

Spatial and temporal frequency. All analyses (in both experiments) were performed on absolute error scores measured in Hz (the absolute value of the difference between the observer's final setting for the temporal frequency of the comparison bar and that of the standard bar). A preliminary analysis of the data revealed that for conditions containing either the highest spatial frequency or the fastest temporal frequency observers were unable to perform the task effectively. Under levels of luminance contrast that were sufficient for the stimuli to be resolved without any difficulty (greater than 4.38%), error magnitude for the spatial frequency of 22.91 cyc/deg was more than 3 standard deviations from the mean of other spatial frequencies. For the temporal frequency of 20 Hz there was a ceiling effect for over-estimation (the maximum over-estimation was 2 Hz), yet observers still performed very poorly (2.88 standard deviations from the mean). As a result, these data were not considered in subsequent analyses. These performance decrements are not unexpected (e.g., [22]).

The first analysis was conducted to separate the contributions of the spatial and temporal frequency components. A 7 (spatial frequency) x 7 (temporal frequency) x 6 (luminance contrast) repeated-measures ANOVA was performed on the absolute error scores. The assumption of non-correlation between repeated measures was checked by calculating the Huynh-Feldt estimate of epsilon (a somewhat less conservative version of the Greenhouse-Geisser estimate). For effects where this assumption was violated the appropriate reduction in degrees of freedom was made to determine the more conservative probability levels that are reported (in all statistical analyses presented for both experiments, including ANOVA effects and post-hoc comparisons). The main effects of luminance contrast, $F(5,40) = 52.12$, $p < 0.0001$, spatial frequency, $F(6,48) = 15.84$, $p < 0.0001$, and temporal frequency, $F(6,48) = 36.84$, $p < 0.0001$ were significant. The interaction effects between luminance contrast and spatial frequency $\underline{F}$ (30,240) = 3.23, $\underline{p} < 0.005$, and luminance contrast, spatial frequency, and temporal frequency, $\underline{F}$ (180,1440) = 1.81, $\underline{p} < 0.03$ were also significant. All other effects were not significant.

The significant main effect of contrast indicates that a minimal level of luminance contrast was required to perform the task accurately. The mean absolute errors were 0.73, 0.66, 0.88, 1.22, 3.18, and 5.27 Hz (descending order of luminance contrast). Supplemental post hoc F-tests indicated that performance for the highest level of luminance contrast (32.45%) was significantly more accurate than performance for the lowest level of contrast (1.13%), but not significantly different from the intermediate level of contrast (4.38%). There were no significant differences between the intermediate and lowest levels. The significance levels for these post hoc tests (and all post hoc tests reported) were determined by dividing the pair-wise type I error rate (.05) by the number of comparisons performed for that effect. The main effect of temporal frequency indicates that performance became worse as the rate of apparent motion increased. The mean absolute errors were 1.48, 1.57, 1.52, 1.87, 1.50, 2.13, and 3.86 Hz (from slowest to fastest rate). Performance for the highest rate (11.33 Hz) was significantly worse than performance for the lowest rate (0.83 Hz) and an intermediate rate (2.00 Hz). There were no significant differences between the intermediate and lowest levels.

The mean errors for the main effect of spatial frequency were 2.99, 2.44, 2.01, 1.73, 1.12, 1.43, and 2.20 Hz (ordered from lowest to highest frequency). Thus, performance was best for intermediate frequencies (in particular 2.86 c/deg), and became worse as spatial frequency increased or decreased. Supplemental F-tests indicated that performance for the intermediate frequency of 2.86 c/deg (1.12) was significantly more accurate than performance for the lowest spatial frequency (0.18 c/deg) and the highest spatial frequency (11.50 c/deg). The difference in performance between the lowest and highest frequencies was also significant. However, the significant interaction effect between spatial frequency and luminance contrast indicated that this pattern changed across levels of contrast. The mean errors for this interaction effect are illustrated in Fig. 4a. When luminance contrast was greater than 2.29% (the four highest levels) the differences in performance across spatial frequency were small. For example, at a luminance contrast of 4.38% there were no significant differences in performance between the intermediate frequency of 2.86 c/deg, the highest frequency, or the lowest frequency. For lower levels of contrast the pattern of performance favoring intermediate frequencies became evident. For example, at a luminance contrast of 2.29% the mean errors between 2.86 c/deg and the highest and lowest frequencies were significantly different. The mean accuracy errors for the highest and lowest frequencies were not significantly different.

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Insert Fig. 4 about here

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The three-way interaction effect indicates that the relationship between spatial frequency and luminance contrast was also dependent upon temporal frequency. At the lower temporal frequencies spatial frequency has little effect, provided that sufficient contrast is present. The pattern of results favoring intermediate spatial frequencies is particularly evident at the intermediate temporal frequencies. At the highest temporal frequency performance is worse in general, and performance differences for spatial frequencies are less marked.

Velocity. A second analysis of error scores was conducted to investigate the joint contribution of spatial and temporal frequency. An independent variable of velocity was computed for each combination of the 7 spatial and temporal frequencies by dividing the temporal frequency of the standard bar by its spatial frequency (see Table 3). A 49 (velocity) x 6 (luminance contrast) repeated-measures ANOVA was conducted on the error scores. The main effects of luminance contrast $F(5,40) = 52.11$, $p < 0.0001$, and velocity $F(48,384) = 7.17$, $p < 0.0001$, were significant. The interaction effect between luminance contrast and velocity $F(240,1920) = 1.96$, $p < 0.01$, was also significant.

Because of the large number of levels for velocity only selected aspects of the results will be presented. The main effect of velocity indicates that, in general, the quality of performance decreased as velocity increased (especially at higher velocities, see Fig. 4b). However, there were also systematic differences related to the specific levels of spatial and temporal frequency, independent of velocity. These are indicated by the recurrent rises and falls in the curve of Fig. 4b. The significant interaction effect between velocity and contrast (not shown) indicates that the pattern of performance for velocity changed across levels of luminance contrast. Both the general trend for decreased performance and the systematic variations due to combinations of specific spatial and temporal components are suppressed at high contrast and accentuated at low contrast.

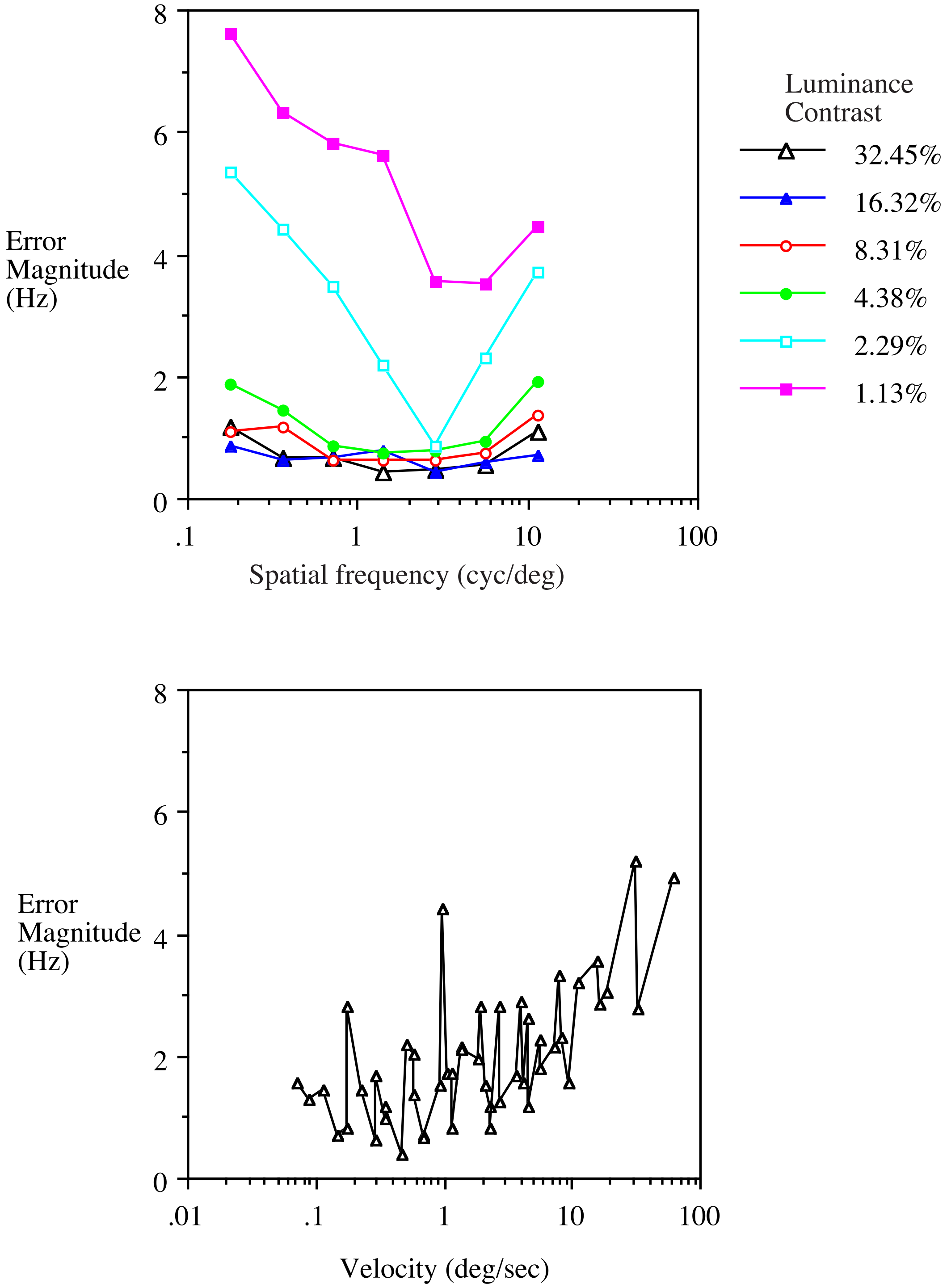

Figure 4. Mean error magnitude (Hz) for Experiment 1. 4a. The interaction effect of spatial frequency and luminance contrast. 4b. The main effect of velocity (averaged across contrast levels).

DISCUSSION

To provide a framework for interpretation a general description of related findings from the basic vision literature will be provided. These studies have typically used different stimuli (sinusoidal wave forms) and tasks (e.g., contrast sensitivity -- adjusting the amount of luminance contrast until a pattern is just discernible). In general, both spatial and temporal frequency are associated with a “bandpass” pattern of contrast sensitivity, with performance peaking at intermediate frequencies and falling off at higher and lower frequencies [14, 22,23,24,25]. Contrast sensitivity for spatial frequency peaks at approximately 3 cycles/degree; contrast sensitivity for temporal frequency peaks at approximately 2 Hz.

This pattern of results was not obtained in the present experiment. For the main effect of temporal frequency a “lowpass” pattern was found: there were no significant differences in performance between the lowest rate (0.83 Hz) and the intermediate rate (2.00 Hz), while performance was degraded significantly for the highest rate (11.33 Hz). A bandpass pattern was found for the main effect of spatial frequency, with significantly better performance at the intermediate spatial frequency of 2.86 c/deg. However, the significant interaction effect of spatial frequency and luminance contrast indicates that this pattern occurred only under low levels of luminance contrast (in particular, 2.29%, see Fig. 4a). With higher levels of contrast there were no significant differences in performance across spatial frequencies.

These results provide partial support for the frequency analysis hypothesis. The rationale for this conclusion will be outlined for spatial frequency (note that the logic applies to temporal frequency as well, e.g., [15]). A lowpass pattern of results, not the typical bandpass pattern, is expected at high levels of luminance contrast. The improved relative performance for low spatial frequencies is expected because these stairstep wave forms will produce higher sinusoidal harmonics that fall in the range of optimal spatial sensitivity (see Table 1a). With sufficiently high levels of luminance contrast the observers could focus on these sinusoidal harmonics to complete the task. Thus, no performance differences are expected between low and intermediate spatial frequencies because in both cases observers are using sinusoidal harmonics that fall in the optimal range of spatial sensitivity. Poor performance is expected for stairstep wave forms with high spatial frequencies because the fundamental and all higher sinusoidal harmonics fall outside the range of maximum spatial sensitivity.

It is also expected that the improved relative performance for low spatial frequencies will be observed only at high levels of luminance contrast. The amplitude (and therefore the perceptibility) of each successive sinusoidal harmonic decreases as luminance contrast is lowered (see Table 1b). For example, the amplitude of the fundamental, first, and second harmonics are 1, 1/2, and 1/4 the amplitude of the stairstep wave form. Thus, the ability of an observer to use the higher sinusoidal harmonics for completing the task will decrease as the luminance contrast of the stairstep wave form approaches threshold levels. As a result, performance is likely to revert to the typical bandpass pattern as luminance contrast decreases.

The results of the experiment provide partial support for these predictions. The bandpass pattern of results did appear for spatial frequency at low levels of luminance contrast (most notably 2.29% contrast, see Figure 4a). Performance for the intermediate frequency of 2.86 c/deg was most resilient to decreases in luminance contrast. This is expected because the fundamental harmonic, which has the highest amplitude (and therefore perceptibility), is centered in the range of maximal spatial sensitivity. Performance for lower spatial frequencies was progressively less resilient: as spatial frequency decreased the amplitude of the spatial harmonics that fell in this range was lowered systematically. The predictions for spatial frequency were not fully supported because a lowpass pattern was not obtained at high luminance contrast. In contrast, a lowpass pattern of results was obtained for temporal frequency. However, the predictions for temporal frequency were not fully supported either, because a bandpass pattern did not appear as luminance contrast was decreased.

Other aspects of these data are also relevant to the hypotheses being considered. Studies in basic vision that have investigated both spatial and temporal frequency have found that velocity (the composite variable) is the critical factor in sensitivity. For example, Kelly [22] found that changes in sensitivity varied with changes in velocity and occurred gradually in an orderly, monotonic fashion. In contrast, the present study found systematic variation in performance that was more closely tied to specific combinations of spatial and temporal frequency than to velocity (see Fig. 4b). This discrepancy is consistent with the frequency analysis hypothesis. The presence of spatial and temporal harmonics, and changes in salience due to luminance contrast virtually ensure that perceived velocity is not a direct function of spatial and temporal frequency.

Experiment 2 was designed to replicate and extend the findings of Experiment 1. The extreme temporal and spatial frequencies in Experiment 1 were not considered. The range of luminance contrast was decreased so that the critical interaction between contrast and frequency could be examined at a finer grain. An important methodological change concerns the initial difference in temporal frequency between the comparison and standard bars. In Experiment 1 only four initial rates were used for the comparison bar, and therefore the average initial error was not constant. This is likely to have produced uncontrolled variation in performance for difficult experimental conditions. Therefore, the average initial error between the two bars was standardized (and lowered) in Experiment 2 (see Table 6).

EXPERIMENT 2

Method

Subjects. The same subjects participated in Experiment 2.

Apparatus. The apparatus was identical to the previous experiment.

Stimuli. Only the six intermediate levels of both spatial and temporal frequency in Table 3 were used in Experiment 2. The perceptual characteristics of the stimuli were measured and analyzed in the same fashion as in Experiment 1 and the results are listed in Table 5. Constant average initial differences between the rates of the standard and comparison bars were maintained (see Table 6).

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Insert Tables 5 and 6 about here

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Procedure. The procedure was the same as the previous experiment, with the exception that testing occurred on five experimental sessions. To summarize, in each session an observer completed 72 trials: a factorial combination of the six temporal frequencies (within-subjects factor) and the six spatial frequencies (within-subjects factor) with two repetitions for each combination. The order of these trials was determined randomly. The luminance contrast was systematically reduced by using stimulus sets 1 through 5 in successive sessions (within-subjects factor).

RESULTS

Spatial and temporal frequency. Final accuracy scores were obtained by averaging across repetitions; a 6 (spatial frequency) x 6 (temporal frequency) x 5 (luminance contrast) repeated-measures ANOVA was performed on these scores. The main effects of luminance contrast, F (4,32) = 14.07, $p < 0.0002$, spatial frequency, F (5,40) = 15.61, $p < 0.0001$, and temporal frequency, F (5,40) = 57.40, $p < 0.0001$, were significant. The interaction effect between luminance contrast and spatial frequency was also significant, F (20,160) = 2.02, $p < 0.04$. All other effects were not significant.

The significant main effect of luminance contrast indicates that a minimal level of luminance contrast was required to perform the task accurately. The mean absolute errors were 0.59, 0.65, 0.70, 1.10, and 1.42 Hz (ordered in descending levels of luminance contrast). Performance for the lowest level of luminance contrast (1.34%) was significantly less accurate than performance for the highest level of contrast (5.70%) and for an intermediate level of contrast (3.26%). Differences in performance between the highest and the intermediate level of contrast was not significantly different. The main effect of temporal frequency indicates that accuracy was best for low and intermediate rates of apparent motion, and worse for high rates. The mean errors were 0.60, 0.62, 0.56, 0.70, 1.13, and 1.75 Hz (ordered from slowest to fastest rate). Accuracy for the fastest rate (11.33 Hz) was significantly worse than accuracy for the slowest rate (1.00 Hz) and an intermediate rate (2.00 Hz). Differences in accuracy for the slowest rate and the intermediate rate were not significant.

The main effect of spatial frequency indicates that accuracy was best for intermediate frequencies, and worse for high and low frequencies. The means were 1.04, 0.93, 0.69, 0.64, 0.73, and 1.32 (ordered from lowest to highest frequency). Supplemental F-tests indicated that performance for the intermediate frequency of 2.86 c/deg was significantly better than performance for the lowest (0.36 c/deg) and the highest frequency (11.50 c/deg). Performance differences for the lowest and highest frequencies was not significantly different. However, the significant interaction effect between spatial frequency and luminance contrast indicated that this pattern of performance was dependent upon contrast. The means for this interaction effect are illustrated in Fig. 5a. As in Experiment 1, the bandpass pattern of performance occurred only at lower levels of luminance contrast. For example, at a luminance contrast level of 2.16% performance for the intermediate frequency of 2.86 c/deg was significantly better than both the lowest frequency (0.36 c/deg) and the highest frequency (11.50 c/deg), while the difference between the highest and lowest frequencies was not significantly different. In contrast to the results of Experiment 1, there were significant differences at low luminance contrast and a lowpass pattern of performance was obtained. For example, at a luminance contrast level of 3.26% there were no significant differences in mean accuracy between the low and intermediate frequencies, while performance for the highest frequency was significantly worse.

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Insert Fig. 5 about here

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Velocity. The six levels of spatial and temporal frequency were used to determine 36 levels of velocity. The final absolute error scores (Hz) were obtained by averaging across repetitions, and a 36 (velocity) x 5 (luminance contrast) repeated-measures ANOVA was conducted. The main effects of luminance contrast $\underline{F}$ (4,32) = 14.06, $\underline{p} < 0.0002$, and velocity $\underline{F}$ (35,280) = 14.47, $\underline{p} < 0.0001$, were significant. The interaction effect $\underline{F}$ (140,1120) =1.59, $\underline{p} < 0.04$, was also significant. A very similar pattern of results were obtained. As in Experiment 1 there was a general trend for the quality of performance to decrease as velocity increased (see Fig. 5b). Again, there were also systematic differences related to the specific levels of spatial and temporal frequency. The significant interaction effect indicates that the pattern of performance changed across levels of luminance contrast. Both the general trend for decreased performance, and the systematic variations due to combinations of specific spatial and temporal components are suppressed at high contrast and emphasized at low contrast.

DISCUSSION

The results of Experiment 2 provide additional support for the frequency analysis hypothesis. A significant interaction revealed that the pattern of performance for spatial frequency was dependent upon luminance contrast. As in Experiment 1, a bandpass pattern for spatial frequency was obtained with low levels of luminance contrast (see Fig. 5a). For the luminance contrast of 2.16% performance for intermediate spatial frequencies was significantly better than performance for both high and low spatial frequencies. In contrast to Experiment 1, the predicted lowpass pattern for spatial frequency was found with high levels of luminance contrast. For the luminance contrast of 3.26% there were no significant differences between the lowest and an intermediate spatial frequency, while performance for the highest spatial frequency was significantly worse.

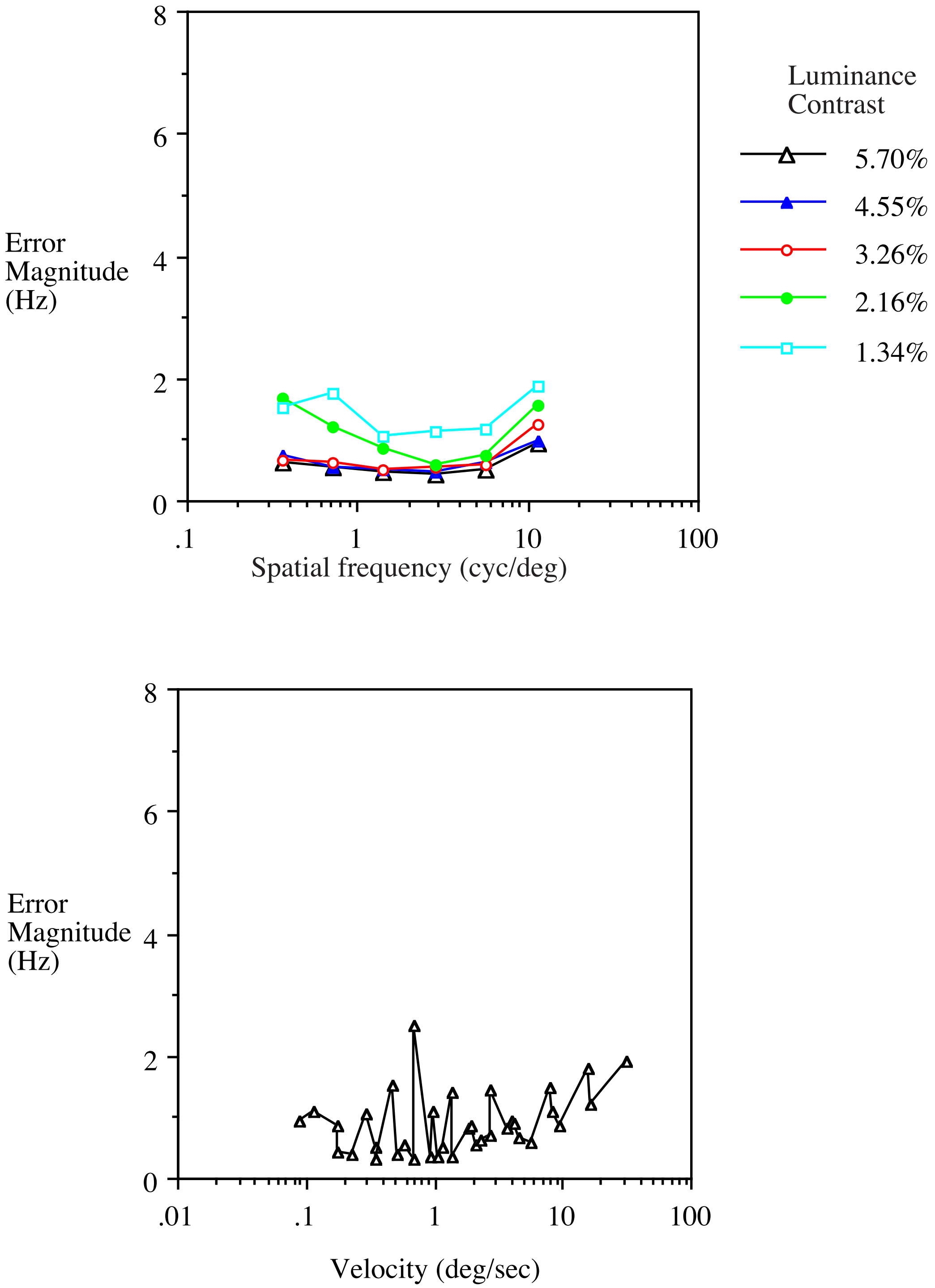

Figure 5. Mean error magnitude (Hz) for Experiment 2. 5a. The interaction effect of spatial frequency and luminance contrast. 5b. The main effect of velocity (averaged across contrast levels).

This pattern of results strongly supports the frequency analysis hypothesis. With sufficient luminance contrast observers could use higher sinusoidal harmonics (falling in the optimal, intermediate range of spatial frequencies) to improve performance for low spatial frequencies. Performance reverted to the bandpass pattern of results when the amplitude of these higher sinusoidal harmonics was reduced by decreases in luminance contrast.

Two additional findings that support the frequency analysis hypothesis were replicated in Experiment 2. As in Experiment 1, performance was more closely tied to specific combinations of spatial and temporal frequency than to velocity (see Fig. 5b). Once again, a lowpass pattern of results, rather than the typical bandpass pattern, was found for temporal frequency. There was no significant difference in performance between the lowest frequency (1.00 Hz) and an intermediate frequency (2.00 Hz), but performance was degraded significantly for the highest temporal frequency (11.33 Hz). Both of these findings deviate from the typical pattern of results obtained in the basic vision literature, but are consistent with the frequency analysis hypothesis.

In fact, the only aspect of the results that is inconsistent with the frequency analysis hypothesis is the finding that performance for lower temporal frequencies did not revert to a bandpass pattern at low levels of luminance contrast. The most likely explanation is that the lowest temporal frequencies chosen in the present experiments were not actually outside the range of optimal sensitivity. In a representative study Kelly [22] found that sensitivity for temporal frequency was best between approximately 0.5 and 5.0 Hz; in contrast, the lowest temporal frequencies investigated in the present experiments were 0.83 Hz (Experiment 1) and 1.00 Hz (Experiment 2). At best, these temporal frequencies were at the low end of the acceptable range. Consider the same comparison for spatial frequency. Kelly found that sensitivity was best from approximately 1.5 to 5.5 cycles/degree. Half of the spatial frequencies tested in Experiments 1 and 2 were below the minimal level. Thus, it is likely that performance for temporal frequency would have reverted to a bandpass pattern at low contrast if lower temporal frequencies had been included.

One potentially confusing aspect of the results needs to be clarified. There are large differences in the absolute levels of performance in Experiments 1 and 2 (compare Figures 4a and 5a). It is likely that a methodological change (the size of the default errors) contributed substantially to these differences. The default error is the difference in temporal frequency between the standard and comparison bar at the beginning of an experimental trial. Differences in the default error rate is especially critical for difficult trials (high/low frequency, low contrast): the perceptual information required to perform the task may be missing or degraded and therefore an observer is less likely to adjust the comparison bar. The average default error rate was 5.40 Hz in Experiment 1; in Experiment 2 the default error rate was lowered to 2.80 Hz and standardized to minimize uncontrolled variation. Although absolute levels of performance in Experiment 2 did not change for easy trials (high contrast, intermediate frequencies), there was a substantial reduction for difficult trials. Of course, it is likely that practice effects did occur between Experiments 1 and 2 (for example, errors are sometimes equal to or higher than the default error rate in Experiment 1 but always lower in Experiment 2). However, it is likely that the decrease in absolute levels of performance on difficult trials was the result of initial error size, not practice effects.

GENERAL DISCUSSION

Perhaps the single most important goal in display design is the mapping between the underlying domain and the visual representation of that domain provided by a graphic display. For animated functional mimic displays this translates into the relationship between the physical rate of flow that exists in the underlying domain and the subjective impression of flow that results from viewing the display. The most prominent example of animated functional mimics in the display literature is the STEAMER system [10,11,12]. The present study and a previous study [2] investigated the effectiveness of that design and found fairly strong evidence that this display configuration does not provide effective mappings between domain and display. From a design perspective the implications are quite serious: at high levels of luminance contrast (those most likely to occur in actual displays) apparent motion can be perceived in the opposite direction and at a different rate than the actual flow that the display is designed to represent.

The present paper provides a potential explanation of why this occurs, using insights derived from Fourier’s theorem and results from the basic visual perception literature. It should be noted that the stimuli used in this study are very different from the stimuli typically used to study the perception of spatial and temporal frequency. For example, spatially extended sinusoidal gratings (with luminance contrast decreasing towards the edges of the stimuli) are usually used to avoid the generation of harmonics. In the present experiment the edges of the narrow "pipe" in which the periodic stimuli moved certainly introduced energy at many spatial and temporal frequencies. Conceivably, this information could have been used to accomplish the rate matching task, or could have produced harmonics that our analysis did not address. Nevertheless, the results seem reasonably consistent with the notion that the harmonics moving in the horizontal direction were responsible for deviations from typical pattern of results.

It is fairly clear that alternative display configurations need to be considered. Bennett and Madigan [28] investigated an alternative design which varied the "contours" of the graphical elements. In one contour condition the graphical elements remained square-shaped (as in Fig. 1); in a second contour condition the sides of the graphical elements became more arrow-shaped with increases in rate-of-flow (e.g., > vs. |). Angled contours were found to improve performance significantly. They produce visual cues that both disambiguate the direction of apparent motion and provide a redundant encoding for rate of flow (the size of the angles). Although angled contours are effective, they also incur a substantial computational cost: each time the underlying rate of flow changes the mimic display must be redrawn completely.

An alternative display design is to incorporate wave forms that approximate a sinusoidal function, rather than a stairstep function (an example is provided in Fig. 6). This design produces harmonics that are far less perceptible, and should therefore produce a more stable representation of the underlying rate of flow. It also has the additional benefit of retaining the computational efficiency associated with color table animation. A preliminary investigation revealed that performance with an approximated sinusoidal wave form was significantly better than performance for a stairstep wave form (both accuracy and latency, under certain experimental conditions). In addition, observers expressed a strong subjective preference for the approximate sinusoidal wave form. It appears that this is a very promising alternative design.

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Insert Fig. 6 about here

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Design guidelines for spatial and temporal frequency

Both the results of the present experiments and the basic vision literature suggest that spatial and temporal frequency are critical factors in the design of animated functional mimics. If wave forms that approximate a sinusoidal function are used (as in Figure 6), then the extensive basic literature can be used to derive design guidelines. Spatial frequencies ranging from approximately

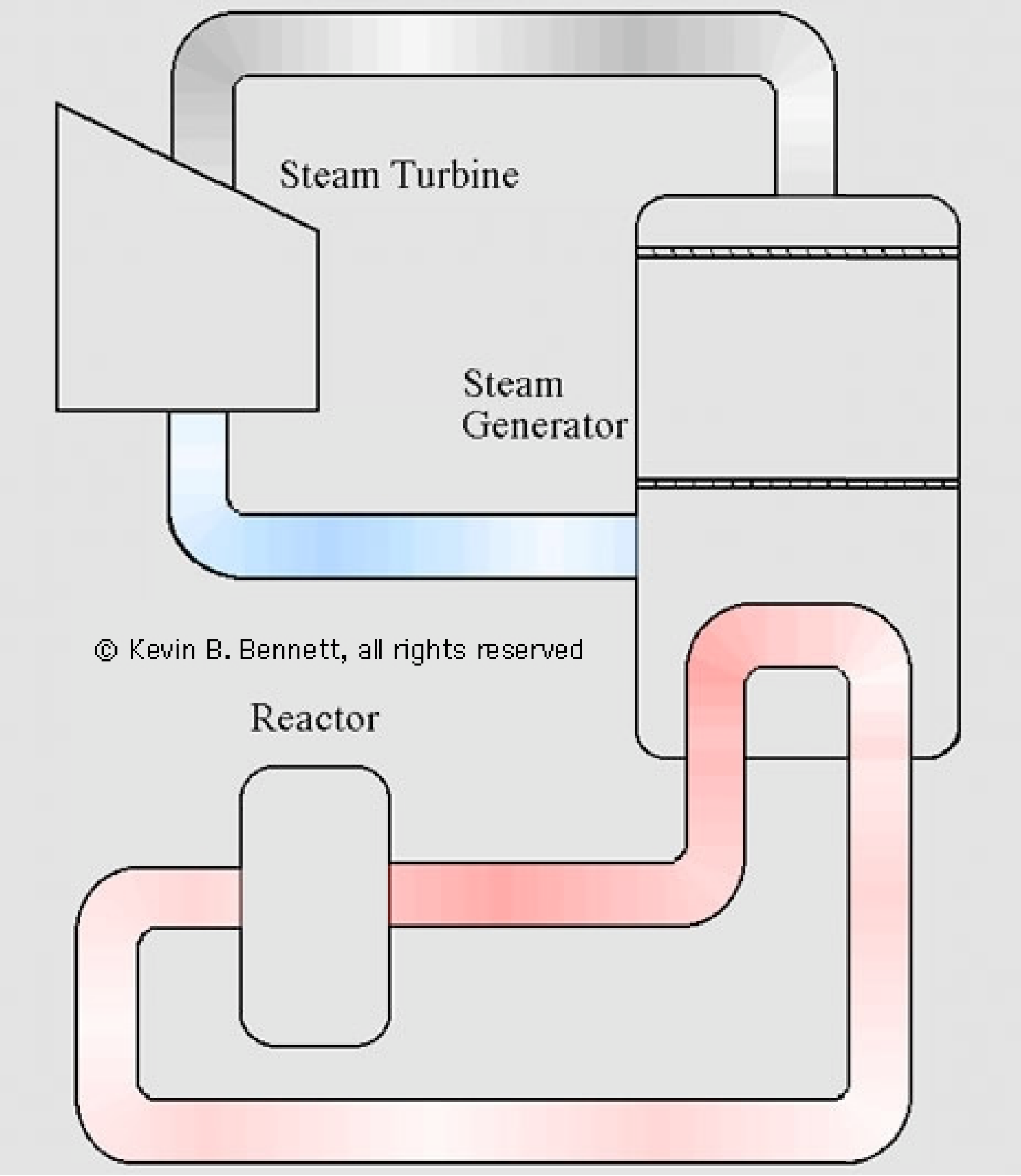

Figure 6. An animated mimic display with an approximate sinusoidal luminance profile.

1.5 to 5.5 cycles/degree and temporal frequencies ranging from approximately 0.5 to 5.0 Hz are recommended. When translating these results into specific design guidelines the ultimate goal of these displays must be considered: to provide an accurate visual representation of the rate of flow that exists in the underlying domain. Therefore, considerations of temporal frequency should take first precedence and the widest acceptable range of temporal frequencies should be used to allow finer discriminations of flow. Thus, when there is no flow in the underlying domain there should be no apparent motion in the display. The lowest possible rate of flow in the domain should be represented by apparent motion with a temporal frequency of 0.5 Hz; the highest possible rate of flow should be represented by apparent motion with a temporal frequency of 5.0 Hz.

It is also the case that temporal frequency cannot be considered independent of spatial frequency (except over a limited range of spatial frequency). A nice feature of the interdependency between these two factors is that within intermediate spatial frequencies performance remains quite high over a broad range of temporal frequencies. For example, Kelly [22] found that a spatial frequency of approximately 3.0 cycles/degree was associated with the highest level of contrast sensitivity for temporal frequencies ranging from approximately 0.5 to 5.0 Hz. Therefore, a spatial frequency of approximately 3.0 cycles/degree is recommended. Choosing a spatial frequency as close as possible to this recommended value becomes particularly important in applied settings because the spatial frequency of a mimic display will change due to head movements, changes in seating position, and ambulatory movements.

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Table 1. Fourier analysis for the stairstep wave forms in Experiment 1. Table 1a: The fundamental spatial frequency (F, measured in cycles per degree) for each of the eight stairstep wave forms used in the experiment is listed in the top row. The first eleven sinusoidal harmonics for each stairstep wave form are listed in the column. Table 1b: The luminance contrast (A -- amplitude, expressed in units corresponding to Michelson's formula, C) for each of the six experimental sessions is listed in the top row. The luminance contrast for the first 11 sinusoidal harmonics is listed in the corresponding column.

1a.

Spatial frequency (cyc / deg)

Stairstep Wave Form:	F	0.18	0.36	0.72	1.43	2.86	5.73	11.50	22.90
Sinusoidal Harmonics:									
Fundamental	F	0.18	0.36	0.72	1.43	2.86	5.73	11.50	22.90
2nd	2 F	0.36	0.72	1.44	2.86	5.72	11.46	23.00	45.80
3rd	3 F	missing							
4th	4 F	0.72	1.44	2.88	5.72	11.44	22.92	46.00	91.60
5th	5 F	0.90	1.80	3.60	7.15	14.30	28.65	57.50	114.50
6th	6 F	missing							
7th	7 F	1.26	2.52	5.04	10.01	20.02	40.11	80.50	160.30
8th	8 F	1.44	2.88	5.76	11.44	22.88	45.84	92.00	183.20
9th	9 F	missing							
10th	10 F	1.80	3.60	7.20	14.30	28.60	57.30	115.00	229.00
11th	11 F	1.98	3.96	7.92	15.73	31.46	63.03	126.50	251.90

1b.

Contrast (C)

Stairstep Wave Form	A	32.45	16.32	8.31	4.38	2.29	1.13
Sinusoidal Harmonics:							
Fundamental	3 A/π	30.99	15.58	7.94	4.18	2.19	1.08
2nd	3/2 A/π	15.49	7.79	3.98	2.09	1.09	0.54
3rd	3/3 A/π	missing					
4th	3/4 A/π	7.75	3.90	1.98	1.05	0.55	0.27
5th	3/5 A/π	6.20	3.12	1.59	0.84	0.46	0.22
6th	3/6 A/π	missing					
7th	3/7 A/π	4.43	2.23	1.13	0.60	0.31	0.15
8th	3/8 A/π	3.87	1.95	0.99	0.53	0.29	0.13
9th	3/9 A/π	missing					
10th	3/10 A/π	3.10	1.56	0.79	0.42	0.22	0.11
11th	3/11 A/π	2.82	1.42	0.72	0.38	0.20	0.10

Table 2. Direction and rate of apparent motion for the sinusoidal harmonics of a stairstep wave form with a selected spatial frequency (0.18 cyc / deg) and contrast level (C = 32.45). A constant update rate (temporal frequency) and a left-to-right shifting of the graphical elements is assumed. For example, relative to the stairstep wave form, the second sinusoidal harmonic has twice the spatial frequency, approximately half the amplitude, and moves at half the velocity in the opposite direction.

	Spatial frequency (cyc / deg)		Contrast (C)		Phase Shift	Apparent Motion	
						Direction	Rate
Stairstep Wave Form:	F	0.18	A	32.45	1/3	→	V
Sinusoidal Harmonics:							
Fundamental	F	0.18	$3\,A/\pi$	30.99	1/3	→	V
2nd	2 F	0.36	$3/2\,A/\pi$	15.49	2/3	←	1/2 V
3rd	3 F	missing					
4th	4 F	0.72	$3/4\,A/\pi$	7.75	4/3	→	1/4 V
5th	5 F	0.90	$3/5\,A/\pi$	6.20	5/3	←	1/5 V
6th	6 F	missing					
7th	7 F	1.26	$3/7\,A/\pi$	4.43	7/3	→	1/7 V
8th	8 F	1.44	$3/8\,A/\pi$	3.87	8/3	←	1/8 V
9th	9 F	missing					
10th	10 F	1.80	$3/10\,A/\pi$	3.10	10/3	→	1/10 V
11th	11 F	1.98	$3/11\,A/\pi$	2.82	11/3	←	1/11 V

Table 3. Velocity (deg / sec) for the spatial and temporal frequencies used in Experiment 1.

		Velocity (deg / sec)							
		Spatial frequency (cyc / deg)							
		0.18	0.36	0.72	1.43	2.86	5.73	11.50	22.90
	0.83	4.62	2.32	1.16	0.58	0.29	0.14	0.07	0.04
	1.00	5.57	2.79	1.40	0.70	0.35	0.17	0.09	0.04
Temporal Frequency (Hz)	1.33	7.41	3.71	1.86	0.93	0.46	0.23	0.12	0.06
	2.00	11.14	5.58	2.79	1.40	0.70	0.35	0.17	0.09
	3.33	18.55	9.30	4.65	2.33	1.16	0.58	0.29	0.15
	6.00	33.42	16.75	8.38	4.19	2.10	1.05	0.52	0.26
	11.33	63.12	31.63	15.83	7.91	3.96	1.98	0.99	0.49
	20.00	111.42	55.84	27.94	13.97	6.99	3.49	1.75	0.87

Table 4. Chromaticity coordinates, luminance, and contrast values for stimuli in Experiment 1.

	Graphical Element	u’	v’	cd / m^2	Contrast (C)
Set 1	1	0.1908	0.4507	62.55	32.45 %
	2	0.1913	0.4521	45.00	
	3	0.1919	0.4532	31.90	
Set 2	1	0.1908	0.4507	62.55	16.32 %
	2	0.1910	0.4514	52.95	
	3	0.1913	0.4521	45.00	
Set 3	1	0.1908	0.4507	62.55	8.31 %
	2	0.1910	0.4512	57.30	
	3	0.1910	0.4514	52.95	
Set 4	1	0.1908	0.4507	62.55	4.38 %
	2	0.1908	0.4511	59.75	
	3	0.1910	0.4512	57.30	
Set 5	1	0.1908	0.4507	62.55	2.29 %
	2	0.1907	0.4510	61.15	
	3	0.1908	0.4511	59.75	
Set 6	1	0.1908	0.4507	62.55	1.13 %
	2	0.1915	0.4499	61.90	
	3	0.1907	0.4510	61.15	

Table 5. Chromaticity coordinates, luminance, and contrast values for stimuli in Experiment 2.

	Graphical Element	u’	v’	cd / m^2	Contrast (C)
Set 1	1	0.1923	0.4525	62.55	5.70%
	2	0.1923	0.4527	59.00	
	3	0.1924	0.4528	55.80	
Set 2	1	0.1923	0.4525	62.55	4.55%
	2	0.1924	0.4527	59.90	
	3	0.1923	0.4527	57.10	
Set 3	1	0.1923	0.4525	62.55	3.26%
	2	0.1924	0.4526	60.04	
	3	0.1923	0.4527	58.60	
Set 4	1	0.1923	0.4525	62.55	2.16%
	2	0.1922	0.4526	60.90	
	3	0.1924	0.4527	59.90	
Set 5	1	0.1923	0.4525	62.55	1.34%
	2	0.1922	0.4525	61.90	
	3	0.1922	0.4526	60.90	

Table 6. Initial temporal frequency (Hz) of standard and comparison bars in Experiment 2.

Comparison Bar	Standard Bar					
1.00	1.80	2.60	3.40	4.20	5.00	5.80
1.33	2.13	2.93	3.73	4.53	5.33	6.13
2.00	2.80	3.60	4.40	5.20	6.00	6.80
3.33	4.13	4.93	5.73	6.53	7.33	8.13
6.00	6.80	7.60	8.40	9.20	10.00	10.80
11.33	12.13	12.93	13.73	14.53	15.33	16.13

LIST OF FIGURES