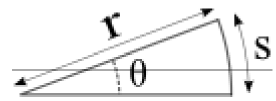
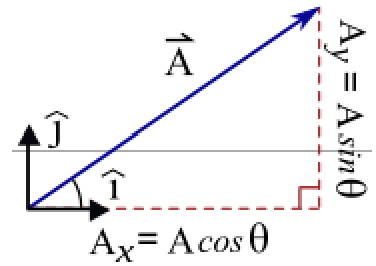
Physics equations/Rewrite

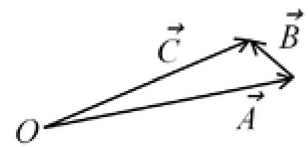
00-Mathematics for this course



Measured in radians, $\theta = s/r$ defines angle (in radians), where s is arclength and r is radius. The circumference of a circle is $C_\odot = 2\pi r$ and the circle's area is $A_\odot = \pi r^2$ is its area. The surface area of a sphere is $A_\bigcirc = 4\pi r^2$ and sphere's volume is $V_\bigcirc = \frac{4}{3}\pi r^3$



A vector can be expressed as, $\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$, where $A_x = A\cos\theta$, and $A_y = A\sin\theta$ are the x and y components. Alternative notation for the unit vectors $(\hat{\imath}, \hat{\jmath})$ include $(\hat{x}, \hat{y})$ and $(\widehat{e_1}, \widehat{e_2})$. An important vector is the displacement from the origin, with components are typically written without subscripts: $\vec{r} = x\hat{x} + y\hat{y}$. The magnitude (or *absolute value* or *norm*) of a vector is is $A \equiv |\vec{A}| = \sqrt{A_x^2 + A_y^2}$, where the angle (or phase), θ, obeys $\tan\theta = y/x$, or (almost) equivalently, $\theta = \arctan(y/x)$. As with any function/inverse function pair, the tangent and arctangent are related by $\tan(\tan^{-1}\mathcal{X}) = \mathcal{X}$ where $\mathcal{X} = y/x$. The arctangent is not a true function because it is multivalued, with $\tan^{-1}(\tan\theta) = \theta \; or \; \theta + \pi$.



The geometric interpretations of $\vec{A} + \vec{B} = \vec{C}$ and $\vec{B} = \vec{C} - \vec{A}$ are shown in the figure. Vector addition and subtraction can also be defined through the components: $\vec{A} + \vec{B} = \vec{C} \Leftrightarrow$ $A_x + B_x = C_x$ AND $A_y + B_y = C_y$

01-Introduction

Text	Symbol	Factor	Exp
giga	G	1 000 000 000	E9
mega	M	1 000 000	E6
kilo	k	1 000	E3
(none)	(none)	1	E0
centi	c	0.01	E−2
milli	m	0.001	E−3
micro	μ	0.000 001	E−6
nano	n	0.000 000 001	E−9
pico	p	0.000 000 000 001	E−12

- 1 kilometer = .621 miles and 1 MPH = 1 mi/hr ≈ .447 m/s
- Typically air density is 1.2kg/m^3, with pressure 10^5Pa. The density of water is 1000kg/m^3.
- Earth's mean radius ≈ 6371km, mass ≈ 6×10^{24} kg, and gravitational acceleration = g ≈ 9.8m/s^2
- Universal gravitational constant = G ≈ 6.67×10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$
- Speed of sound ≈ 340m/s and the speed of light = c ≈ 3×10^8m/s
- One light-year ≈ 9.5×10^{15}m ≈ 63240AU (Astronomical unit)
- The electron has charge, e ≈ 1.6×10^{-19}C and mass ≈ 9.11×10^{-31}kg. 1eV = 1.602×10^{-19}J is a unit of energy, defined as the work associated with moving one electron through a potential difference of one volt.
- 1 amu = 1 u ≈ 1.66×10^{-27} kg is the approximate mass of a proton or neutron.
- **Boltzmann's constant** = $k_B \approx 1.38 \times 10^{-23}$ J K^{-1} , and the **gas constant** is R = $N_A k_B \approx 8.314$ J K^{-1} mol^{-1}, where $N_A \approx 6.02 \times 10^{23}$ is the Avogadro number.

- $k_e = \dfrac{1}{4\pi\varepsilon_0}$ ≈ 8.987× 10^9 N·m²·C^{-2} is a fundamental **constant of electricity**; also $\varepsilon_0 = \dfrac{1}{4\pi k_e}$ ≈ 8.854 × 10^{-12} F·m^{-1} is the vacuum permittivity or the electric constant.
- μ_0 = 4π × 10^{-7} NA ≈ 1.257 × 10^{-6} N A (magnetic permeability) is the fundamental constant of magnetism: $\sqrt{\varepsilon_0 \mu_0} = 1/c$.
- $\hbar$ = h/(2π) ≈ 1.054×10^{-34} J·s the reduced Planck constant, and $a_0 = \dfrac{\hbar^2}{k_e m_e e^2}$ ≈ .526 × 10^{-10} m is the Bohr radius.

03-Two-Dimensional Kinematics

$$x = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x \Delta t^2 \qquad v_x = v_{0x} + a_x \Delta t \qquad v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

$$y = y_0 + v_{0y}\Delta t + \frac{1}{2}a_y \Delta t^2 \qquad v_y = v_{0y} + a_y \Delta t \qquad v_y^2 = v_{x0}^2 + 2a_y \Delta y$$

$v^2 = v_0^2 + 2a_x \Delta x + 2a_y \Delta y$ ...in advanced notation this becomes $\Delta(v^2) = 2\vec{a} \cdot \Delta\vec{\ell}$.

In free fall we often set, a_x=*0* and a_y= -*g*. If angle is measured with respect to the x axis:

$v_x = v\cos\theta \qquad v_y = v\sin\theta \qquad v_{x0} = v_0\cos\theta_0 \qquad v_{y0} = v_0\sin\theta_0$

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not needed 2nd semester

06-Uniform Circular Motion and Gravitation

- $2\pi\ rad = 360\ deg = 1\ rev$ relates the radian, degree, and revolution.
- $f = \dfrac{\#\,\text{revs}}{\#\,\text{secs}}$ is the number of revolutions per second, called **frequency**.
- $T = \dfrac{\#\,\text{secs}}{\#\,\text{revs}}$ is the number of seconds per revolution, called **period**. Obviously $fT = 1$.
- $\omega = \dfrac{\Delta\theta}{\Delta t}$ is called **angular frequency** (ω is called omega, and θ is measured in radians). Obviously $\omega T = 2\pi$
- $a = \dfrac{v^2}{r} = \omega v = \omega^2 r$ is the acceleration of **uniform circular motion**, where v is speed, and r is radius.
- $F = G\dfrac{mM}{r^2} = mg^*$ is the force of gravity between two objects, where G ≈ 6.674 × 10^{-11} m^3·kg^{-1}·s^{-2}.

07-Work and Energy

- $KE = \dfrac{1}{2}mv^2$ is kinetic energy, where *m* is mass and *v* is speed..

- $U_g = mgy$ is gravitational potential energy,where y is height, and $g = 9.80\,\frac{m}{s^2}$ is the gravitational acceleration at Earth's surface.
- $U_s = \frac{1}{2}k_s x^2$ is the potential energy stored in a spring with spring constant k_s.
- $\sum KE_f + \sum PE_f = \sum KE_i + \sum PE_i - Q$ relates the final energy to the initial energy. If energy is lost to heat or other nonconservative force, then Q>0.
- $W = F\ell\cos\theta = \vec{F}\cdot\vec{\ell}$ (measured in Joules) is the work done by a force F as it moves an object a distance ℓ. The angle between the force and the displacement is θ.
- $\sum \vec{F}\cdot\Delta\ell$ describes the work if the force is not uniform. The steps, $\Delta\vec{\ell}$, taken by the particle are assumed small enough that the force is approximately uniform over the small step. If force and displacement are parallel, then the work becomes the area under a curve of F(x) versus x.
- $P = \frac{\vec{F}\cdot\vec{\Delta}\ell}{\Delta t} = \vec{F}\cdot\vec{v}$ is the power (measured in Watts) is the rate at which work is done. (***v*** is velocity.)

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not needed second semester

18-Electric charge and field

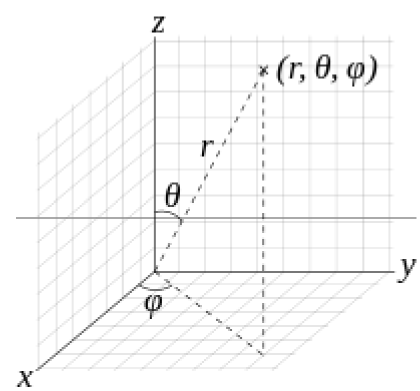
- $F = k_e\frac{qQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}$ is Coulomb's law for the force between two charged particles separated by a distance r: $k_e \approx 8.987\times10^9$ N·m²·C^{-2}, and $\varepsilon_0 \approx 8.854\times10^{-12}$ F·m^{-1}.
- $\vec{F} = q\vec{E}$ is the electric force on a "test charge", q, where $E = k_e\frac{Q}{r^2}$ is the *magnitude* of the electric field situated a distance *r* from a charge, Q.

Consider a collection of N particles of charge Q_i, located at points $\vec{r}_i$ (called *source points*), the electric field at $\vec{r}$ (called the *field point*) is:

- $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{\widehat{\mathcal{R}}_i Q_i}{|\vec{\mathcal{R}}_i|^2} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{\vec{\mathcal{R}}_i Q_i}{|\vec{\mathcal{R}}_i|^3}$ is the electric field at the field point, $\vec{r}$, due to point charges at the source points,$\vec{r}_i$, and $\vec{\mathcal{R}}_i = \vec{r} - \vec{r}_i$, points from source points to the field point.

CALCULUS supplement:

$\vec{E}(\vec{r}) = k_e\int\frac{\hat{\mathcal{R}}dQ}{\mathcal{R}^2}$ is the electric field due to distributed charge, where $dQ \to \lambda d\ell \to \sigma dA \to \rho dV$, and (λ, σ, ρ) denote linear, surface, and volume density (or charge density), respectively.



Cartesian coordinates (x, y, z). Volume element: $dV = dxdydz$. Line element: $d\vec{\ell} = \hat{x}dx + \hat{y}dy + \hat{z}dz$. Three basic area elements: $\hat{n}dA = \hat{z}dxdy$, or,$\hat{x}dydz$, or, $\hat{y}dzdx$.

Cylindrical coordinates (ρ, φ, z): Volume element: $dV = \rho\,dr\,d\varphi\,dz$. Line element: $d\vec{\ell} = \hat{\varphi}rd\varphi + \hat{r}dr + \hat{z}dz$. Basic area elements: $\hat{n}dA = \rho\,d\varphi\,dz\,\hat{\rho}$ (side), and, $\rho\,d\rho\,d\varphi\,\hat{z}$ (top end).

Spherical coordinates (r, θ, φ): Volume element: $dV = r^2\,dr\,\sin\theta\,d\theta\,d\varphi \to 4\pi r^2 dr$ (if symmetry holds). Line element:$d\vec{\ell} = \hat{r}\,dr + \hat{\theta}\,r\,d\theta + \hat{\varphi}\,r\,\sin\theta\,d\varphi$. Basic area element of a sphere: $\hat{r}dA = \hat{r}\,r^2 d\Omega$, where $d\Omega$ is a solid angle.

19-Electric Potential and Electric Field

- $U = qV$ is the potential energy of a particle of charge, q, in the presence of an electric potential V.
- $\Delta V = -E\ell\cos\theta = \vec{E}\cdot\vec{\ell}$ (measured in Volts) is the variation in electric potential as one moves through an electric field E. The angle between the field and the displacement is θ. The electric potential, V, decreases as one moves parallel to the electric field.
- $\Delta V = -\sum \vec{E}\cdot\Delta\ell$ describes the electric potential if the field is not uniform.
- $V(\vec{r}) = k\sum\frac{Q_j}{\mathcal{R}_j}$ due to a set of charges Q_j at $\vec{r}_j$ where $\vec{\mathcal{R}}_j = \vec{r} - \vec{r}_j$.
- $Q = CV$ is the (equal and opposite) charge on the two terminals of a capacitor of capicitance, C, that has a voltage drop, V, across the two terminals.
- $C = \varepsilon A/d$ is the capacitance of a parallel plate capacitor with surface area, A, and plate separation, d. This formula is valid only in the limit that d^2/A vanishes. If a dielectric is between the plates, then $\varepsilon > \varepsilon_0 \approx 8.85\times10^{-12}$ due to shielding of the applied electric field by dielectric polarization effects.
- $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$ is the energy stored in a capacitor.
- $u = \frac{\varepsilon}{2}E^2$ is the *energy density* (energy per unit volume, or Joules per cubic meter) of an electric field.

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calculus supplement not needed for exams

- $V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}}\vec{E}\cdot d\ell$ in the limit that the Riemann sum becomes an integral.
- $\vec{E} = -\vec{\nabla}V$ where $\vec{\nabla} = \hat{x}\partial/\partial x + \hat{y}\partial/\partial y + \hat{z}\partial/\partial z$ is the del operator.
- $\varepsilon_0\int\vec{E}\cdot\overrightarrow{dA} = Q_{encl}$ is Gauss's law for the surface integral of the electric field over *any* closed surface, and Q_{encl} is the total charge inside that surface.
- $\int\vec{D}\cdot\overrightarrow{dA} = Q_{free}$ is a useful variant if the medium is dielectric. ***D***=ε***E*** is the electric displacement field. The **permittivity**, $\varepsilon = (1+\chi)\varepsilon_0$, where $\varepsilon_0 \approx 8.85\times10^{-12}$, and the electric susceptibility, χ, represents the degree to which the medium can be polarized by an electric field. The free charge, Q_{free}, represents all charges except those represented by the susceptibility, χ.

Help with Gauss' Las

The quiz at Special:permalink/1391093 is about a cylinder or a sphere. There are actually two cylinders, one a long wire, and the other a flat plane not unlike that of a parallel plate capacitor. It relates a surface integral to a volume integral:

$$\varepsilon_0\underbrace{\int\int}_{area}\vec{E}\cdot d\vec{A} = \underbrace{\int\int\int}_{volume}\rho\,dV = Q_{enclosed}$$

1. The first is a surface integral of $\vec{E}\cdot\vec{n}$ over a closed and the second is a volume integral of the charge density over the volume enclosed by this "closed surface". A "closed surface" is any surface without a "hole": A balloon is an open surface unless you tie off the exit, in which case it is closed.
2. The "outward" unit normal $\hat{n}$ can only be defined for a closed surface. We shall call our closed surface a "Gaussian Surface" (GS). Gauss' Law holds for any GS that you can imagine. But it is only useful if the integral is easy to solve.

3. To use Gauss' Law, arrange for the surface to coincide with the "field point", which is defined as the point where you want to calculate the field. We shall only use this law in cases where the integrand is uniform over either all or part of the area. Also, we restrict ourselves to cases where the charge density ρ is uniform. With these simplifications, the integral form of Gauss' law becomes an algebraic expression: $\varepsilon_0 EA^* = V^*\rho$.
4. We define small-r r to represent the location of the field point, and big-R R to represent the object. This works in spherical coordinates and in cylindrical coordinates if the cylinder is a long wire of radius R. For this wire, $r < R$ if the field point is inside the wire, and $r > R$ if the field point is outside the wire.
5. In cylindrical coordinates for a "pancake", $R \to \infty$, and the field point is z for a wire of length $H >> R$. The field point is inside the object if $z < H/2$ and outside the object if $z > H/2$.

Sphere: $V^* = \frac{4}{3}\pi r^{*3}$ $\quad A^* = 4 = \pi r^{*2}$ $\quad$ (note that $A = dV/dr$)

- If the field point is inside the object, then both r values are the same and equal to the field point: $r^* = r$
- If the field point is outside the object, then $r^* = r$ for the surface integral (calculation of A^*) but the smaller value of $r^* = R$ for the calculation of the volume integral A^*.

Long wire: H>>R: $V^* = H\ \pi r^{*2}$ $\quad A^* = H\ 2\pi r^*$ (V is height × footprint; A is height × circumference.)

(Note that the circumference of a circle is the derivative of its area. Note also that E does not depend on the length of the wire H because that term cancels on both sides.)

- If the field point is inside the object, then both r^* values are the same and equal to the field point: $r^* = r$
- If the field point is outside the object, then $r^* = r$ for the surface integral (calculation of A^*) but the smaller value of $r^* = R$ for the calculation of the volume integral A^*.

Flat pancake:H<<R: This problem is tricky, because the pancake has two areas (circles). The electric field is parallel to the unit normal because it points away from positive charge. Hence the area used is twice the area of a circle. If the GS is situated inside the pancake, the volume also doubles, because our GS covers a volume extending from $-z$ to $+z$ (in other words, the "cylinder" is 2z long if z<H/2, but it is H long if z>H/2). We include a dagger on the two $(2^\dagger)$ as a reminder that we need to double certain numbers since the GS is a cylinder situated at the origin (extending from −z to +z) with two circular areas at each end.

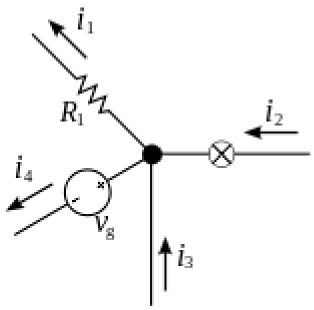
$V^* = H\ \pi R^{*2}$ (outside object) .. or .. $V^* = \left(2^\dagger z\right)\ \pi R^{*2}$ (inside object) $\quad A^* = 2^\dagger \pi R^2$

20-Electric Current, Resistance, and Ohm's Law

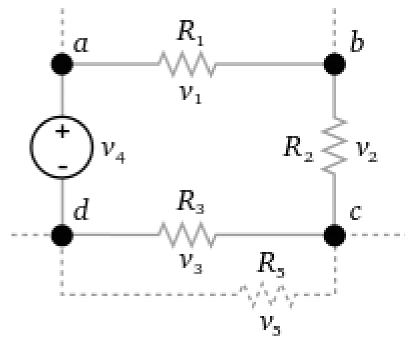
- $I = \dfrac{\mathrm{d}Q}{\mathrm{d}t}$ defines the electric current as the rate at which charge flows past a given point on a wire. The direction of the current matches the flow of positive charge (which is opposite the flow of electrons if electrons are the carriers.)
- $V = IR$ is Ohm's Law relating current, I, and resistance, R, to the difference in voltage, V, between the terminals. The resistance, R, is positive in virtually all cases, and if R > 0, the current flows from larger to smaller voltage. Any device or substance that obeys this linear relation between I and V is called ohmic.
- $I = nqAv_{drift}$ relates the density (n), the charge(q), and the average drift velocity (v_{drift}) of the carriers. The area (A) is measured by imagining a cut across the wire oriented such that the drift velocity is perpendicular to the surface of the (imaginary) cut.
- $R = \dfrac{\rho L}{A}$ expresses the resistance of a sample of ohmic material with a length (L) and area (A). The 'resistivity', ρ ("row"), is an intensive property of matter.
- Power is energy/time, measured in joules/second or J/s. Often called P (never p). It is measured in watts (W)

- Current is charge/time, measured in coulombs/second or C/s. Often called I or i. It is measured in amps or ampheres (A)
- Electric potential (or voltage) is energy/charge, measured in joules/coulomb or J/C. Often called V (sometimes E, *emf*, $\mathcal{E}$). It is measured in volts (V)
- Resistance is voltage/current , measured in volts/amp or V/A. Often called R (sometimes r, Z) It is measured in Ohms (Ω).
- $P = IV = I^2 R = \frac{V^2}{R}$ is the power dissipated as current flows through a resistor

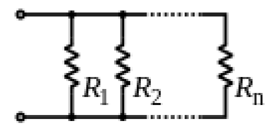
21-Circuits, Bioelectricity, and DC Instruments



The current entering any junction is equal to the current leaving that junction. $i_2 + i_3 = i_1 + i_4$



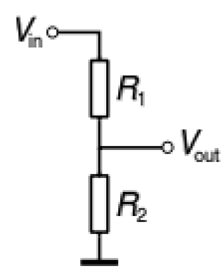
The sum of all the voltages around the loop is equal to zero. $v_1 + v_2 + v_3 - v_4 = 0$



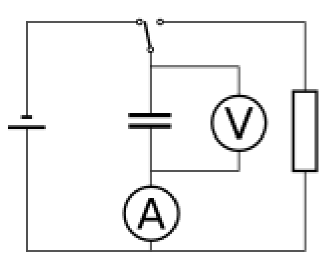
Resistors in parallel

Resistors in series

- $\sum_{k=1}^{n} I_k = 0$ and $\sum_{k=1}^{n} V_k = 0$ are Kirchoff's Laws[3]
- $V_{\text{out}} = \frac{R_2}{R_1 + R_2} V_{\text{in}}$ for the voltage divider shown.



voltage divider



In this example, we assume that the rectangular element is a resistor, R, and that the internal resistance of the voltage source (not shown) is also R. The ammeter and voltmeter shown are ideal.

- **Simple RC circuit**[4] The figure to the right depicts a capacitor being charged by an ideal voltage source. If, at t=0, the switch is thrown to the other side, the capacitor will discharge, with the voltage, *V* , undergoing exponential decay:

$$V(t) = V_0 e^{-\frac{t}{RC}} \ ,$$

where V_0 is the capacitor voltage at time $t = 0$ (when the switch was closed). The time required for the voltage to fall to $\frac{V_0}{e} \approx .37 V_0$ is called the RC time constant and is given by

$$\tau = RC \ .$$