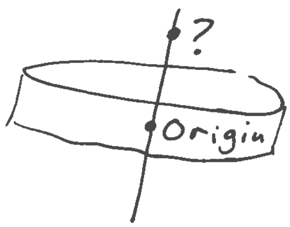
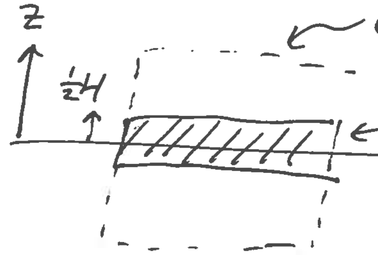
pe 19 using Gauss Law $\varepsilon_0 \int \vec{E} \cdot d\vec{A} = Q_{enc}$ Version A

$A_{\circ} = \pi r^2$ (circle) $C_{\circ} = 2\pi r$ (circle) $A_{\text{cyl}} = 2\pi r h$ (side area)

(1)



Gaussian Surface is cylinder of height z and radius R



$Q_{enc} = V_{cyl} \cdot \rho$

$\rho V_{cyl} = \underbrace{\pi R^2 \cdot H}_{\text{Volume}} \rho = Q_{enc}$

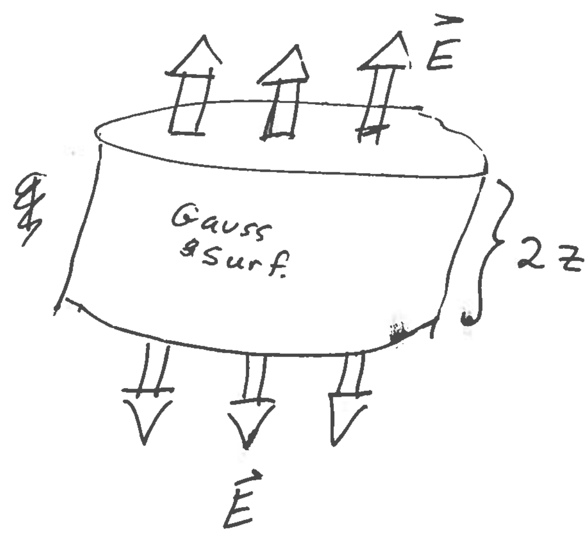
Now do $\oint \vec{E} \cdot d\vec{A} = \underbrace{\oint_{side}}_{\text{zero}} + \underbrace{\oint_{ends}}_{2 \cdot \pi R^2 \cdot E}$

$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = Q_{enc}$

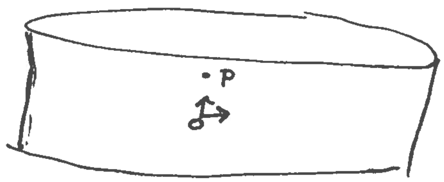
$\varepsilon_0 \, 2\pi R^2 E = \pi R^2 H \rho$

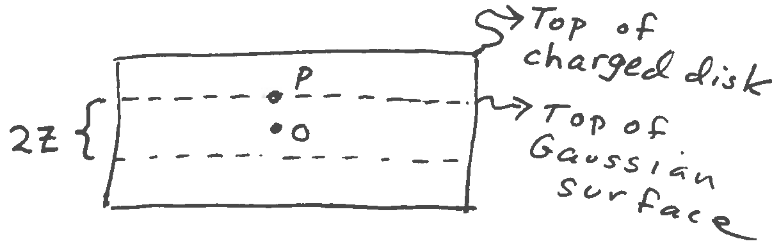
$2 \varepsilon_0 E = H \rho$

$$\boxed{E = \frac{1}{2\varepsilon_0} H \rho}$$



2 Field point inside disk





Hint: Always put your Gaussian surface so that the field point (i.e. where you seek to know $\vec{E}$) is on the Gaussian surface.
Also: If the integral $\oint \vec{E}\cdot d\vec{A}$ is not "easy", you probably can't find E using this method.

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = Q_{enc} = \rho \cdot Vol.$$

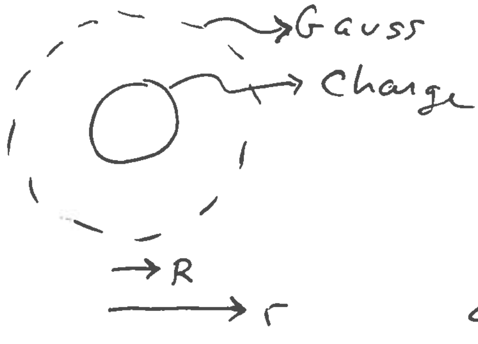
$$\underbrace{2}_{\text{Top + Bottom}}\varepsilon_0 \pi R^2 E = \rho \pi R^2 \cdot \underbrace{2Z}_{\text{Height of Gaussian surface}}$$

Height of Gaussian surface. You can't use H b/c the other charge is not enclosed.

$$\varepsilon_0 E = \rho Z$$

$$\boxed{E = \frac{\rho Z}{\varepsilon_0}}$$

(3) $r > R \rightarrow$ Gauss Surface larger than charged sphere.



$A_{sphere} = 4\pi r^2$ (Gauss)

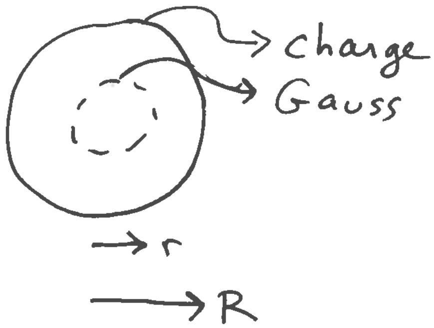
$V_{sphere} = \frac{4}{3}\pi R^3$ (charge)

$$\varepsilon_0 \oint_{Gauss} \vec{E}\cdot d\vec{A} = Q_{enc} = \rho \cdot \text{Volume}$$

$$\varepsilon_0 \underbrace{4\pi r^2}_{\text{area}} E = \underbrace{\frac{4}{3}\rho \cancel{\pi} R^3}_{\text{Vol} \times \rho} \rightarrow \boxed{E = \frac{R^3}{3\varepsilon_0}\frac{\rho}{r^2}}$$

$$\boxed{\varepsilon_0 r^2 E = \rho \frac{R^3}{3}}$$ $\leftarrow$ as per quiz

(4) $r < R \rightarrow$ Gauss surface inside charged sphere



$$\varepsilon_0 \oint_{Gauss} \vec{E}\cdot d\vec{A} = 4\pi r^2 \varepsilon_0 E$$

$$Q_{enc} = \int_{Gauss} \rho \, dV = \frac{4\pi}{3} r^3 \rho$$

(the other charge (between r and R) is not enclosed by the Gauss surface)

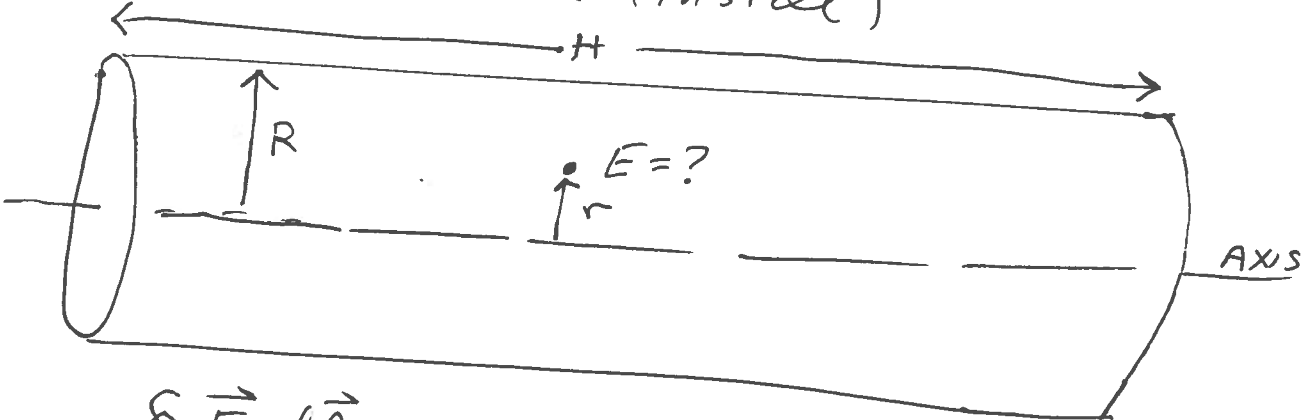
$$\therefore \quad 4\pi r^2 \varepsilon_0 E = \frac{4\pi}{3} r^3 \rho \rightarrow \boxed{\varepsilon_0 E = \frac{1}{3} r \rho}$$

or on quiz:

$$\boxed{r^2 \varepsilon_0 E = \frac{1}{3} r^3 \rho}$$

⑤ Long wire of radius R

Want E at $r < R$ (inside)



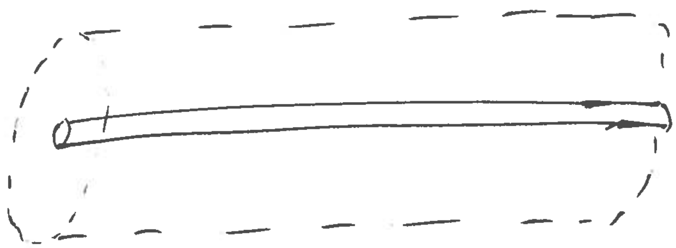
$$\underset{Gauss}{\oint} \vec{E}\cdot d\vec{A} = E \cdot \underbrace{2\pi r \cdot H}_{Area}$$

H is length of wire

$$\int_{enclosed} \rho\, dV = \rho \cdot \underbrace{\pi r^2 \cdot H}_{Vol.}$$

$$\varepsilon_0 E \cdot 2\pi \cancel{r}\, H = \rho \cdot \pi \cancel{r}^2 H \Rightarrow \boxed{\varepsilon_0 E = \frac{1}{2}\rho r}$$

⑥ Want E at $r > R$ (outside)



$$\underset{Gauss}{\oint} \vec{E}\cdot d\vec{A} = 2\pi r H E$$

$$\int_{enclosed} \rho\, dV = \pi R^2 H \rho$$

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = \int \rho\, dV$$

$$\boxed{2\varepsilon_0 r E = R^2 \rho}$$