pe Surface Integrals CALC pe19 version A

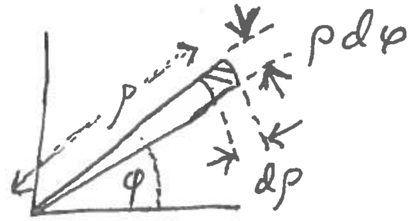
Review $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z = AB\cos\theta$

If $\hat{e}_i$ and $\hat{e}_j$ are unit vectors: $\hat{e}_i \cdot \hat{e}_j = 0$ if $i \neq j$

$d\vec{A} = \hat{n}\, dA$ where $\hat{n}$ = outward unit normal $= 1$ if $i = j$

and dA = area element.

Use polar coordinates to find dA at ends of cylinder

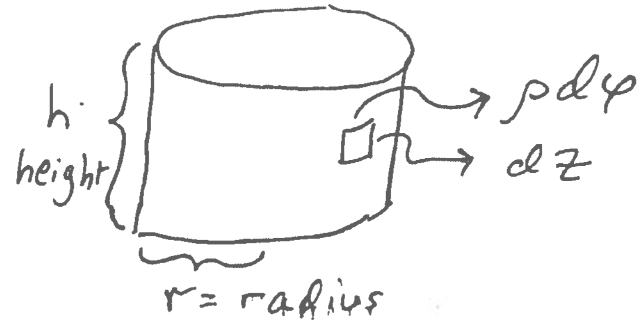


$dA = \rho\, d\rho\, d\varphi$ (end)

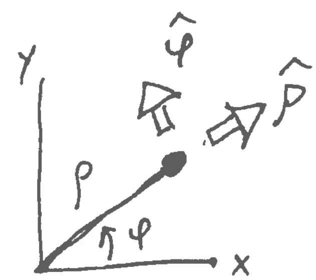
φ goes from 0 to 2π

Make 3-D sketch to find dA on the side:

$dA = \rho\, d\varphi\, dz$



The $\hat{\rho}$ $\hat{\varphi}$ $\hat{z}$ unit vectors point along direction of increasing value of the variable



$\odot\, \hat{z}$ is out of paper

This $\oplus$ denots into paper.

For this cylinder:

$$\boxed{\begin{aligned} &\text{Top: } d\vec{A} = \rho\, d\rho\, d\varphi\, \hat{z} \\ &\text{Side: } d\vec{A} = \rho\, d\varphi\, dz\, \hat{\rho} \end{aligned}}$$

version A

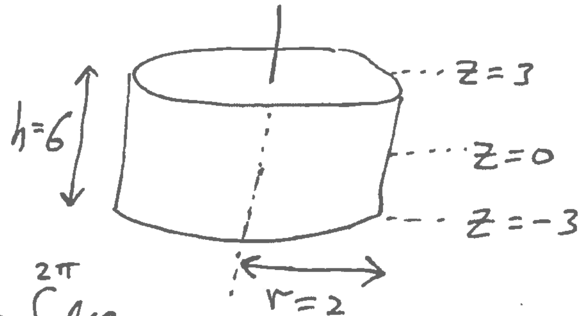
②
scurf int

① $\vec{F} = (2.03 + 1.29z)\rho^2 \hat{\rho} + 8.35 z^3 \hat{z}$

Top: $\hat{n} = \hat{z}$

$\vec{F} \cdot \hat{n} = 8.35 z^3 = 8.35(3^3)$

$= 225.45$



$$\int_{Top} \vec{F} \cdot \hat{n}\, dA = 225.45 \int_0^2 \rho\, d\rho \int_0^{2\pi} d\varphi$$

$$= (225.45)\left(\tfrac{1}{2} 2^2\right)(2\pi)$$

$$= 2833 = \boxed{2.8E3 = \int_{TOP} \vec{F} \cdot d\vec{A}}$$

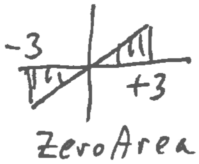
② $\vec{F} \cdot \hat{\rho} = (2.03 + 1.29z)\rho^2$

this will go away

$$\int_{SIDE} \vec{F} \cdot d\vec{A} = \underbrace{\int_{-3}^{3} dz \int_0^{2\pi} d\varphi\, \rho}_{dA} \underbrace{\left\{(2.03 + 1.29z)\rho^2\right\}}_{\vec{F} \cdot \hat{\rho}}$$

$$= \int_{-3}^{3} dz \int_0^{2\pi} d\varphi \left(2.03 \rho^3\right)$$ where we drop the term with $1.29z$.

Drop term because



$\int_{-3}^{3} z\, dz = 0$ since $f(z) = z$ is odd function, i.e., $f(z) = -f(-z) \rightarrow$ integral vanishes

Here $\rho = r = 2$

$$\int_{SIDE} \vec{\mathcal{F}}\cdot d\vec{A} = 2.03\,(2^3)\underbrace{(6)}_{\int_{-3}^{3}dz}\underbrace{(2\pi)}_{\int_0^{2\pi}d\varphi} = (2.03)(8)(12\pi)$$

$$= \boxed{612 = \int_{side} \vec{\mathcal{F}}\cdot d\vec{A}}$$

(3) $\oint \vec{\mathcal{F}}\cdot d\vec{A} = \int_{TOP} \vec{\mathcal{F}}\cdot d\vec{A} + \int_{side} \vec{\mathcal{F}}\cdot d\vec{A} + \int_{Bottom} \vec{\mathcal{F}}\cdot d\vec{A}$

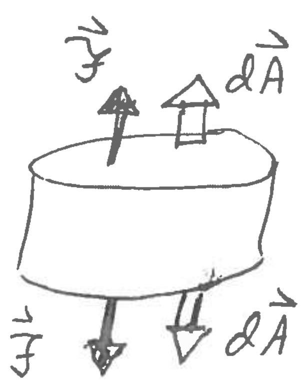
→ The circle on the integral sign denotes the integral over the entire (closed) surface.

The integral over the bottom is like the integral over the bottom: $\vec{\mathcal{F}}\cdot\hat{z} = 8.35\, z^3 = \mathcal{F}_z$ ← z^{th} component

Since z^3 is an odd power, $\mathcal{F}_z(-z) = -\mathcal{F}_z(+z)$.

MAKE A SKETCH!

$$\underbrace{\vec{\mathcal{F}}\cdot d\vec{A}}_{Top} = \underbrace{\vec{\mathcal{F}}\cdot d\vec{A}}_{Bottom}$$



Because $\mathcal{F}_z$ is odd the net flux is outward on top + bottom

$$\oint \vec{\mathcal{F}}\cdot d\vec{A} = 2\int_{TOP}(\ldots) + \int_{SIDE}(\ldots) = 2(2833) + 612 = \boxed{6278 = \oint \vec{\mathcal{F}}\cdot d\vec{A}}$$

Be Careful: If $\vec{\mathcal{F}}\cdot\hat{z} = 8.35\, z^2$, then $\mathcal{F}_z(-z) = \mathcal{F}_z(+z)$ and $\int_{TOP}(\ldots) = -\int_{Bottom}(\ldots)$. In that case

$\boxed{\oint \vec{\mathcal{F}}\cdot d\vec{A} = \int_{SIDE}(\ldots)}$ → See, for example, Version B.