~~Uniform~~ Harmonic Oscillators (SHO)

Next Wikiquiz:

$$\frac{d}{dx} f(g(x)) = \frac{df}{dg}\frac{dg}{dx}$$

This will be on formula sheet:

$$\frac{d}{d\theta}\sin\theta = \cos\theta \qquad \frac{d}{d\theta}\cos\theta = -\sin\theta$$

1. Find $\frac{d}{dt}\cos\omega t$ Answer: $\theta = \omega t$

Let $f(t) = \cos(\omega t)$

~~$\frac{d}{d\theta}\cos\theta = \frac{d\cos\theta}{d\theta} = -\sin\theta$~~

$$\frac{d}{dt} f(t) = \frac{df}{d\theta}\frac{d\theta}{dt} = \frac{d}{d\theta}(\cos\theta)\frac{d(\omega t)}{dt}$$

$$= -\sin(\omega t)\cdot\omega \quad \text{since } \frac{d}{dt}(\omega t) = \omega$$

$$\boxed{= -\omega\sin\omega t}$$

2. Find $\frac{d^2}{dt^2}\cos\omega t = \frac{d}{dt}\left(\frac{d}{dt}\cos\omega t\right)$

$$= \frac{d}{dt}\left(-\omega\sin\omega t\right) = -\omega\frac{d}{dt}\sin\omega t = \boxed{-\omega^2\cos\omega t}$$

Hmm: If $x(t) = A\cos\omega t$

$$\frac{d^2x}{dt^2} = -A\omega\cos\omega t = -\omega^2 x$$

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x} \quad \text{if} \quad x = A\cos\omega t$$

$$= B\sin\omega t$$

$$= A\cos\omega t + B\sin\omega t$$

$$= A\cos(\omega t - \varphi)$$

Hooke's Law:

$$F = -kx$$

k = spring constant

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

Newton's II Law for mass-spring:

$$F = -kx = ma = m\frac{d^2x}{dt^2}$$

$$\boxed{\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x}$$ ~ ordinary differential equation that is linear

$$x = A\cos\omega t + B\sin\omega t \text{ where } \omega^2 = \frac{k}{m}$$