10 - dynamics

1) 36 m \[ \text{WANT } \alpha \]

Recall: \( S = \Theta r = \) arclength
\( V = \omega r = \) tangential speed
\( \alpha = \frac{d^2 \theta}{dt^2} = \) tangential acceleration
Also recall:
\[ \alpha = \omega \dot{\omega} \]
\[ \alpha = \frac{\omega^2}{r} = \frac{36}{6.8 \cdot 26} = 20.4 \, \text{s}^2 = \alpha \]

2) Use \( I = \sum m_j r_j^2 = MR^2 \)
Since \( r_j = R \) so that \( I = R^2 \sum m_j \)
Here \( M = 2.2 \, \text{kg} \) and \( r = 0.57 \, \text{m} \)
\[ I = (2.2)(0.57)^2 = 0.715 \, \text{kg} \cdot \text{m}^2 = I \]

3) Want \( KE \) at 1.7 rev/sec
\[ \omega = \frac{1.7 \, \text{rev}}{\text{sec}} \cdot \frac{2\pi \, \text{rad}}{1 \, \text{rev}} = 3.4\pi \, \text{s}^{-1} \]

3) continued: \( KE = \frac{1}{2} I \omega^2 = \frac{(0.715)(3.4)^2\pi^2}{2} \]
\[ = 40.8 \, J = KE \]
I know the units are Joules b/c energy is Joules.
I could also get the units from
\[ [I] = [\text{kg} \cdot \text{m}^2] \quad \text{and} \quad [\omega]^2 = \text{s}^{-2} \]
\[ \frac{1}{2} I \omega^2 = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \]
\[ \quad \Rightarrow \text{since } [\omega] = \text{s}^{-1} \]

4) \( I = \sum m_j r_j^2 = \sum m_r^2 + \sum m_r^2 = I_1 + I_2 \)
\[ I = \frac{1}{2} MR^2 \quad \text{for solid disk} \]
\[ R = \frac{D}{2} \]
\[ I = \frac{1}{2}(M_1 R_1^2 + M_2 R_2^2) = \frac{M}{2} \left( \frac{D_1^2}{4} + \frac{D_2^2}{4} \right) \]
\[ = \frac{M}{8}(D_1^2 + D_2^2) \quad \text{where} \quad D_1 = 0.46 \, \text{m} \\
M_1 = M_2 = M = 3.8 \, \text{kg} \quad D_2 = 0.9 \, \text{m} \]
\[ \tau = F \cdot R = \frac{FD}{2} = \frac{(76)(0.46)}{2} = \tau \]
\[ = (76)(0.23) \, \text{newton-meters} \]
\[ \tau = I \alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{(76)(0.23)(8)(0.46^2 + 0.9^2)}{3.8} \]
\[ \alpha = 36.05 \, \text{rad}^2/\text{sec}^2 \]