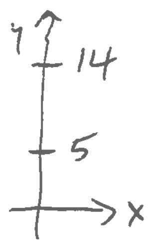
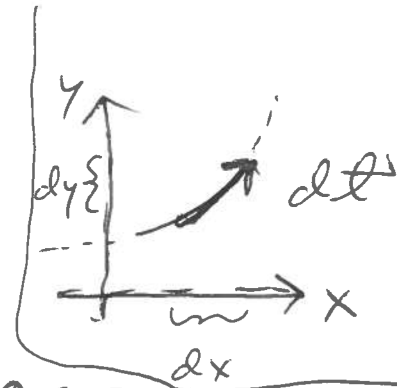
07 - line Integral - version A

① 

$\vec{F}\cdot d\vec{\ell} = (F_x\hat{x} + F_y\hat{y})\cdot(\hat{x}\,dx + \hat{y}\,dy)$

$= F_x\,dx + F_y\,dy$



path on $x=0$: $\vec{F}(0,y) = 9\cdot 0\cdot y\,\hat{x} + 9.5y^3\hat{y}$

$= 9.5y^3\hat{y}$ along y-axis

$= F_y\hat{y}$ (since $F_x = 0$)

$$\int_{path}\vec{F}\cdot d\vec{\ell} = \int_{path}(9.5)y^3\,dy$$

$$= 9.5\int_5^{14} y^3\,dy = \frac{9.5}{4}y^4\Big|_5^{14} = \frac{9.5}{4}\left(14^4 - 5^4\right) = 89754$$

$\boxed{= 8.98E4}$

(since $\int y^3\,dy = \frac{1}{4}y^4 + \text{const}$)

② 

$d\vec{\ell} = \underbrace{r\,d\theta}_{ds}\,\hat{\theta}$ $\vec{F} = r^7\theta^9\hat{r} + r^7\theta^5\hat{\theta} \rightarrow \vec{F}\cdot d\vec{\ell} = r^8\theta^5$

$\theta = 0 \rightarrow \frac{\pi}{2}$ (quarter circle)

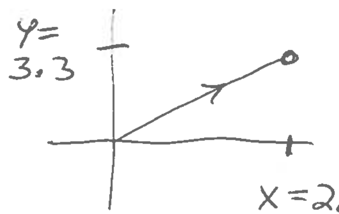
$$\int\vec{F}\cdot d\vec{\ell} = \int_0^{\pi/2} r^8\theta^5\,d\theta = R^8\frac{\theta^6}{6} = 8^8\frac{(\pi/2)^6}{6} = \boxed{4.2\times 10^7}$$

(Radius = 8)

(3) $\vec{F} = 4xy\,\hat{x} + 7.7x\,\hat{y}$

$d\vec{\ell} = \hat{x}\,dx + \hat{y}\,dy$

$\vec{F}\cdot d\vec{\ell} = 4xy\,dx + 7.7x\,dy$



$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{3.3}{2.5}$

path is $y = mx$ or $x = m^{-1}y$

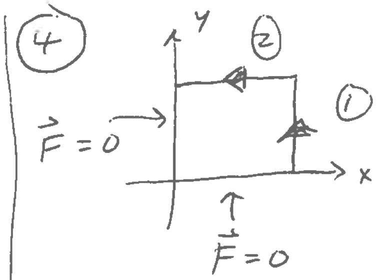
$$\int \vec{F}\cdot d\vec{\ell} = \int_0^{2.5} 4x(mx)\,dx + \int_0^{3.3} 7.7(m^{-1}y)\,dy$$

$$= 4\left(\frac{3.3}{2.5}\right)\int_0^{2.5} x^2\,dx + 7.7\left(\frac{2.5}{3.3}\right)\int_0^{3.3} y\,dy$$

$$= 4\left(\frac{3.3}{2.5}\right)\left(\frac{2.5^3}{3}\right) + 7.7\left(\frac{2.5}{3.3}\right)\frac{3.3^2}{2}$$

$$= \boxed{59.3}$$

(4)



$\vec{F} \neq 0$ only along paths (1) + (2)

PATH 1 $\quad d\vec{\ell} = \hat{y}\,dy$

$\vec{F} = -x^2y^2\,\hat{x} + x^2y^3\,\hat{y}$

$$\int_{(1)} \vec{F}\cdot d\vec{\ell} = \int_{(1)} x^2y^3\,dy \quad (1)\ @\ x=1$$

$$\boxed{\int_{(1)} \vec{F}\cdot d\vec{\ell} = \int_0^1 y^3\,dy = \frac{1}{4}}$$

PATH 2 $\quad d\vec{\ell} = \hat{x}\,dx$ where $dx < 0$

$$\int_{(2)} \vec{F}\cdot d\vec{\ell} = \int_{x=1}^{x=0} F_x\,dx$$

$$\int_{(2)} \vec{F}\cdot d\vec{\ell} = \int_1^0 (-x^2y^2)\,dx \quad @\ y=1$$

$$\boxed{\int_{(2)} \vec{F}\cdot d\vec{\ell} = \int_0^1 x^2\,dx = \frac{1}{3}}$$

closed path (around loop) $\rightarrow \oint \vec{F}\cdot d\vec{\ell} = \frac{1}{4} + \frac{1}{3} = \frac{7}{12} = \boxed{.583}$