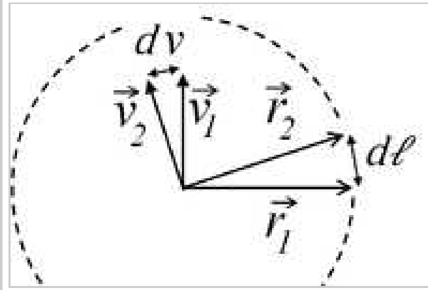
Geometrical proof of equation for uniform circular motion

The figure depicts a change in the position and velocity of a particle during a brief time interval Δt. The distance traveled is



uniform circular motion (here the Latin *d* was used instead of the Greek *Δ*

1. Define $\Delta\ell = |\vec{r}_2 - \vec{r}_1|$, and $\Delta v = |\vec{v}_2 - \vec{v}_1|$
2. $\Delta\ell = v\Delta t$ (rate times time equals distance).
3. $\Delta\vec{v} = \vec{a}\Delta t$ (definition of acceleration).
4. $\Delta v = a\Delta t$ (taking the absolute value of both sides).
5. $\frac{\Delta v}{v} = \frac{\Delta\ell}{r}$ (by similar triangles). Substituting (2) and (4) yields:
6. $\frac{a\Delta t}{v} = \frac{v\Delta t}{r}$, which leads to $\frac{a}{v} = \frac{v}{r}$, and therefore:
7. $a = \frac{v^2}{r}$