

- 3.1 Notational issues with the displacement (\mathbf{r}) vector
- 3.2 Rigorous discussion of acceleration along a curved path
- 4 Derivation of Kepler's third law from the inverse square law

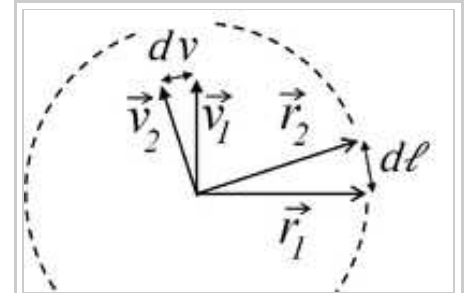
Geometrical proof of equation for uniform circular motion

The figure depicts a change in the position and velocity of a particle during a brief time interval Δt . The distance traveled is

1. Define $\Delta \ell = |\vec{r}_2 - \vec{r}_1|$, and $\Delta v = |\vec{v}_2 - \vec{v}_1|$
2. $\Delta \ell = v \Delta t$ (rate times time equals distance).
3. $\Delta \vec{v} = \vec{a} \Delta t$ (definition of acceleration).
4. $\Delta v = a \Delta t$ (taking the absolute value of both sides).
5. $\frac{\Delta v}{v} = \frac{\Delta \ell}{r}$ (by similar triangles). Substituting (2) and (4)

yields:

6. $\frac{a \Delta t}{v} = \frac{v \Delta t}{r}$, which leads to $\frac{a}{v} = \frac{v}{r}$, and therefore:
7. $a = \frac{v^2}{r}$



uniform circular motion (here the Latin d was used instead of the Greek Δ)