A fingerboard intonation chart for the violin and viola

Guy Vandegrift and Michelle Smith
Wright State University

Author Note

Guy Vandegrift and Michelle Smith
Wright State University, Lake Campus

Correspondence concerning this article should be addressed to Guy Vandegrift, Wright State University Lake Campus, Celina, OH 45822. Email: guy.vandegrift@wright.edu

Abstract

Intonation tests using double-stops played to just consonance can be used by violin and viola students to check virtually any note students might play. Fingerboard charts display the discrepancy between just and equal tempered tuning in a way that offers visual cues as to the distinction between these intonations. Just intervals that are more difficult for students to hear can be constructed using the perfect fourth, fifth, and major sixth. Confidence-building exercises are introduced so that students may verify their ability to construct these just intervals.

A Fingerboard Intonation Chart for the Violin and Viola

Testing intonation requires both a reference pitch, as well as a means for gauging one's ability to match that pitch. While an electronic tuner provides the necessary feedback, such devices can reduce the joy of practicing. Burns (1999) explains how just intervals correspond to the frequency ratios {3/2, 4/3, 5/3, 5/4, 6/5, 8/5}. To this list we add two "compound" ratios {5/2, 10/3} in order to construct intonation tests. This use of just intervals is no better than students' ability to play them in tune. For that reason, we introduce procedures for constructing difficult intervals from three easily recognized ones (perfect fourth and fifth, and major sixth). The fingerboard charts of Figures 1 and 2 differ from those of the 18th century (Barbieri 2008) in that the spacings have been carefully calculated for equal tempered tuning of the open strings. While many students tune strings to just open fifths, fingerboard charts for such tuning would be hopelessly complicated. Beginning and intermediate students could use either just or equal tempered open string tuning with only a modest loss of precision. All activates described here have been tested on a viola and a 3/4 sized violin. Passages that include a cello part were tested on a full sized cello.

Two groups of students might benefit from these exercises: Those with a keen interest in mathematics are apt to find this subject so fascinating that they explore these exercises at great length, improving their intonation in the process. Another group of students, namely those who spend long hours practicing, might also benefit from performing exercises that can be performed long after artistic alertness has been dulled by too much practice. These exercises are "mechanical" in that musical taste is not involved. Ross (2004) introduces what he calls "melodic tuning", which is entirely subjective. But Ross also proposes "mechanical" exercises such as the use of a finger to briefly stop a resonating but un-played open string.

“Large” and “Small” Errors

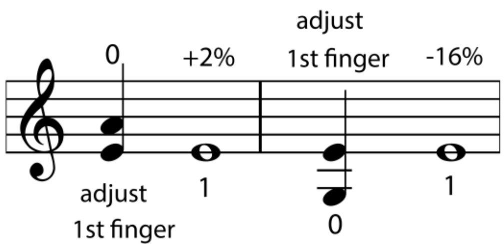
In order to remove unwanted clutter from charts and passages, the % symbol is used to denote “cent” (1 semitone = 100 cents = 100%). Loosen (1993) pointed out that accomplished violinists routinely deviate by 8% from the tempered scale, and we shall refer to any deviations from equal temperament as an “error”, with the full understanding in the hands of an artist, such errors may be deliberate. It is also convenient to label errors as “small” or “large”. As discussed in the (optional) appendix, each factor of {3} in the frequency ratio adds a “small” error of approximately 2%. With the exception of one 6% error, all “small” errors in the fingerboard charts are 4% or less. The fingerboard charts also depict “large” errors that involve a factor of {5} in the frequency ratio. These “large” errors are either 14% or 16% on the fingerboard chart (assuming equal tempered open strings). If open strings are tuned to just fifths, most “large” errors equal 21.5% (the *syntonic comma*). Students should strive to learn this “large” adjustment in pitch, even if they cannot distinguish between the various (14%-21.5%) adjustments. All but the most advanced students could view all “large” errors as a comma-like adjustment of 18%±4%. Such precision exceeds that of most commonly available electronic tuning devices. If we accept this tolerance of 4%, then all “large” errors can be treated as roughly equal, and can be understood using two simple rules:

- Just major thirds and sixths are “narrow”
- Just minor thirds and sixths are “wide

“The just minors are too wide” makes for a good mnemonic if one imagines overweight coal miners who have just cause for refusing to work because they cannot fit through escape tunnels.

Testing Notes with Just Intervals

We introduce the conventions of the violin and viola fingerboard charts with a pair of just interval tests for the first-finger E♮ on the "D" string (Vandegrift, 1994). In the passages that follow, quarter and half notes do not indicate tempo. The quarter notes are to be held (or repeated) until just consonance is achieved, and the whole note denotes the final result:

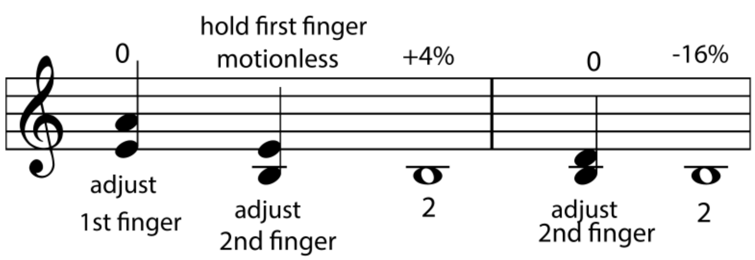


(1)

Students who consistently tune both intervals to just consonance with sufficient precision to discern the pitch difference are ready to begin the activities associated this paper. Passage (1) introduces student to this "comma-like" pitch adjustment, which is 18% for equal tempered open strings and 21.5% for strings tuned to just fifths. To build confidence in making this adjustment, the following activity challenges students to achieve it without using a double stop: Tune the first-finger E♮ against an adjacent string, and then flatten or sharpen the note BEFORE playing the double stop that verifies consonance with the other string. Incidentally, the second bar of (1) yields an equal tempered perfect fourth with an error of only 2%. Constructions of other approximate equal tempered consonant intervals are described later in this paper.

The fingerboard charts of figures 1 and 2 depict the two versions of this first-finger E♮ as large blue or red circles. These circles are clustered around white and gray circles that represent finger location for all twelve notes of the equal tempered scale (white denotes naturals and gray denotes sharps and flats). The right side of the fingerboard charts display the notes and fingerings for these tests. The notes for passage (1) are described in the chart as "adjacent open string" tests, either against a "higher" or a "lower" string. To save space, only one test is shown on the chart. For example, only the "D" strings E♮ is shown for the violin and only the "G" string's A♮ is shown for the viola. Similar tests on adjacent strings are omitted.

The vertical and horizontal displacements of the blue and red circles on the fingerboard charts convey information about pitch error, as well as which open string begins the interval test. Circles shifted upward represent a longer stopped string and correspondingly lower pitch. Likewise, circles shifted downward indicate an intonation test yielding a pitch sharper than the tempered pitch. If the string did not stretch as it was pressed to the fingerboard, the circles would depict exact finger locations for achieving just consonance. The reference string is defined as the first (open) string used to test the note, and the circles are displaced horizontally towards this reference string. For example, in the passage that follows, small blue circles and large blue circles are both shifted to the right. This is because the reference string is "A" for the first bar, "D" for the second bar, and both are situated to the right of the "G" string used to play the B♮:



(2)

The first bar in (2) is discussed on a website by Sassmannshaus (2012), and is labeled in the chart as a "multiple 4ths from higher string". Such multi-step tests are more difficult because it is nearly impossible to adjust one finger while the other remains motionless. They must be practiced slowly and carefully, training both fingers to land lands properly, which after all, is how the instrument should be played.

The fingerboard charts of figures 1 and 2 offer intonation tests for nearly every note on the lower octave of each string. Different strings will sound the same pitch at different timbres (for example, B♮ can be played on the “A” in 1st position or on the “D” in 3rd position.) The ability to independently test the same pitch played on different strings might serve to enhance the student’s confidence in distinguishing between pitch and timbre.

Advise on using the charts

It is not easy to look at the collection of colored circles and squares and find the corresponding passage on the right side of a fingerboard chart. For this reason, it is recommended that students first learn the chart’s passages. In doing so, it is important to also practice passages that are absent from the chart because an equivalent passage is already listed (on an adjacent string). For example, the small blue square indicating a test for the violin’s G♮ on the “E” string does not have a corresponding passage on the chart. Instead, an equivalent passage for the violin’s C♮ on the “A” string is displayed. This passage is labeled as “fourth/octave lower string (6ths)”. Here, (6ths) refers to the fact that the same test can be achieved using a combination of major and minor sixths. Note how the cancelation of two “large” errors in these major and minor sixths results only a “small” error.

Methods to strengthen the student’s ability to hear and play intervals to just consonance will be given in later sections. Some of these activities are best performed on an instrument tuned to just open fifths, while tempered fifths are preferred for other activities. Many commercially available electronic tuners have a precision of ±2.5% (or equivalently a range of 5% that the tuner registers as the same pitch). For the student seeking to gain confidence in tuning just fifths, a student could use the electronic tuner to tune highest open string. Then, tune the lower three strings by ear to perfect just fifths. When tested against the electronic tuner, the lowest string should be 6% flat, an error well within the range of an electronic tuner (which is presumably tuned to tempered fifths). Next, sharpen that lowest string until it registers zero error. Finally, without using the electronic tuner, tune by ear the two middle strings ("D" and "A" on the violin). Strive to make all fifths equally dissonant. Success at this endeavor is indicated by checking all four strings against the electronic tuner. This success will give the student confidence in his or her ear, and the activity does not need direct supervision by a teacher.

Testing Notes with Harmonics

The fingerboard charts show five harmonics for each string, but only a few are useful as intonation tests. Each harmonic must be identified with two locations on the fingerboard chart: A diamond (◆) indicates where the finger lightly touches the string, while a cross (✚) identifies the pitch actually sounded by the harmonic. This pitch actually sounded by a harmonic is identified by situating the cross (✚) where one would press the finger (hard against fingerboard) in order to play the note. The vertical location of a cross indicates whether the harmonic sounds sharp or flat.

The h4 harmonic can serve as a useful intonation test when played on the instrument's lowest string. On the violin it tests G♮ on the "E" string (or C♮ on the viola's "A"). While this note can also be tested using the instrument's lowest open string, some students might find it easier to initially test notes that are unison (using h4) until they gain confidence in recognizing notes separated by one or more octaves.

Harmonics can also be played as shifting exercises to keep the left hand relaxed and in proper position. An excellent warm-up exercise involves playing h2 on the instruments lowest string, and shifting down to first position to play h3 through h6, using the second finger to play both h5 and h6 (an exercise in finger independence). Students might enjoy playing the bugle tunes Taps and Reveille, as well as the theme song from Spielberg's movie, *Close Encounters of the Third Kind*. The harmonic shifting exercise also motivates students to hold the instrument properly. Many folk fiddle players know that there is no need to hold the violin properly if one remains in first position (especially if the fourth finger is never used). The fact that proper hand position is not needed for the repertoire used by beginning students, can lead some to wonder why they can't hold the instrument any way they wish. Young people often exhibit two personality traits that adults find both refreshing and frustrating: They like to (1) think for themselves, and (2) set lofty goals. While first trait makes them apt to invent their own way of holding the instrument, the second trait can be used to keep the first at bay: Students who wish to shift like a virtuoso will practice the harmonic shifting exercises and therefore be forced to hold the violin properly.

Another useful exercise is to play h2 on one string and h3 on the adjacent lower string, in fourth position. The resulting pitches should be identical (if the strings are not too worn.)

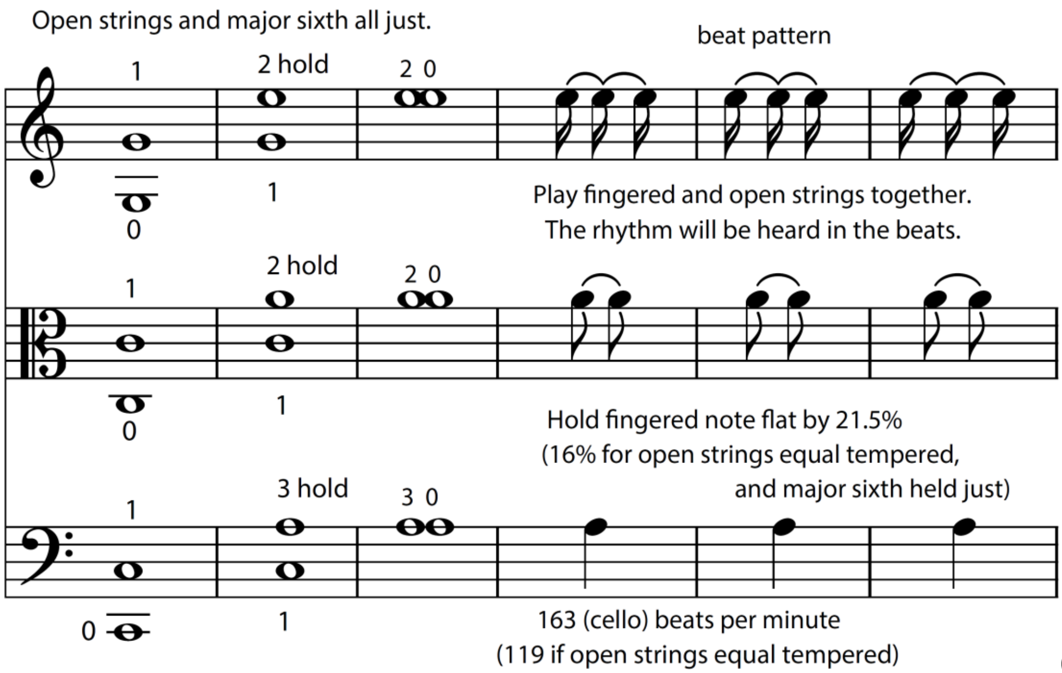
Constructing and Testing Just Intervals

Previous sections have focused on testing a given note's pitch. Now we seek to enhance the precision of such tests with activities designed to help students play intervals to just consonance. One activity involves the construction of difficult intervals using more basic intervals. Beginning with *a priori* mastery of the unison {1/1}, octave {2/1}, perfect fifth {3/2} and fourth {4/3}, every just interval used by the fingerboard charts can be constructed. In these activities it is best to tune the open strings to just fifths. To the extent that the open strings and intermediate intervals at exact just consonance, the constructed interval will be at exact just consonance (i.e., there is no "error.) It is important that students first master the basic intervals (unison, octave, fifth, fourth, sixth). Sundberg (1973) pointed out that even the octave must be learned, since the ear tends to prefer “stretched” (i.e., slightly wide) octaves.

As students learn to play a given intervals to just consonance, confidence-building activities can be introduced. Most confidence-building activities use just intervals to bring students close to just consonance, i.e., to within the comma-like “large” error of 14%-22%. Then, the student listens for this adjustment as the interval is brought to just consonance. In the days of analog radio, listeners “hunted” for the optimal setting of the dial as they delicately nudged it through the correct broadcast frequency, seeking the best reception. String players often tune instruments in the same fashion. The ability to reduce the range over which one must “hunt” greatly facilitates this search, because if the range is too wide, students are apt to “hunt” either too quickly or too slowly. “Hunting” too quickly causes them to pass through consonance without noticing it. “Hunting” too slowly leaves them waiting in vain for an already consonant interval to suddenly improve. An example of “hunting” through a narrow range occurred in passage (1), where students need only “hunt” downward by less than 22% to find the just major sixth.

Enhancing Confidence in the Just Major Sixth

A simple confidence-building activity involving the just major sixth has already been discussed. A far more difficult activity is suitable only for those who enjoy an extraordinary challenge. Beats between nearly equal pitches are used to assess the intonation of a fingered note created using just intervals:



(3)

Observe in (3) that the highest fingered note is flat, and listen for a beat pattern when played against the highest open string. The tempo of these beats will depend on whether the open strings are just or equal tempered fifths. With just-tuned open strings, the tempo will be 163 beats per minute for the cello, with the viola and violin beating twice and thrice as fast, respectively. This passage was played by an amateur violist on all three instruments. Most listeners heard clear and pleasant beats on the cello, but the two higher instruments made annoying dissonant sounds that could be vaguely perceived as possessing beats. An amateur conductor of local musicals and high school orchestras listened to the beats on the violin and viola, and claimed to hear a beat pattern roughly consistent with the theoretical tempo.

To verify that the beats were actually present, the beat pattern for each instrument was measured electronically. We used readily available technology: A Radio Shack tie clip microphone was fed into the sound recorder application found on a Windows PC at a sample rate 44100 Hz. When a passage by the violin was replayed at exactly half the sample rate (i.e. down one octave), it made clearly identifiable beats. When the viola's signal was played at half speed, the beat pattern was sporadic, sometimes seeming to disappear altogether. The recorded signals were also analyzed by Matlab to produce the spectrograms shown in Figure 3. The four red horizontal patterns show four harmonics for each instrument, with the fundamental appearing at the bottom. Note that all four harmonics exhibited beats, with the second harmonic beating at twice the fundamental's beat frequency, with the third harmonic beating at thrice that frequency. This hierarchy of beat frequencies is exactly as one would expect. Figure 3 also documents an absence of beats in the first portion of the viola's signal, as well as a weak signal at the fundamental (which would explain the absence of beats). The spectrograph allowed us to count the beats and establish the tempo on each instrument. The results of a typical trial run are shown below:

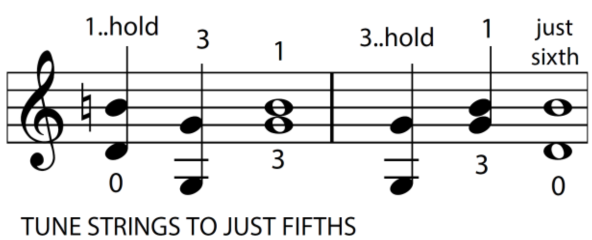
Instrument	Tempo (bars/min)	Tempo Error	Fingered Pitch Error (%)
"ideal"	163	0	-21.5%
violin	109	-54	-14.4%
viola	132	-31	-17.4%
cello	169	+6	-22.3%

Here, the "ideal" tempo refers to the number of bars each instrument plays per minute (one beat per bar for the cello, two for the viola, and three for violin). Since an attempt was made to tune to just open fifths, the fingered pitch should be 21.5% flat. In this effort, the largest error occurred for the violin, which beat triplets at a tempo of 109 beats per minute. An absence of beats is seen in the first two seconds of the viola's spectrogram, and the presence of beats at the second harmonic suggests that the viola has a tendency to not sound at the fundamental harmonic at this pitch. The fingered pitch was calculated using the observed beats. Here, all three instruments achieved the proper (flattened) pitch to within 6%.

Back in 1965, when music seemed to play a more prominent role in public education, students in a middle school orchestra played a bow-control game in which they held a bowed note as long as possible without changing direction. A far more challenging (virtually unwinnable) game would be for a string quartet to attempt to achieve the rhythm of beats in passage (3).

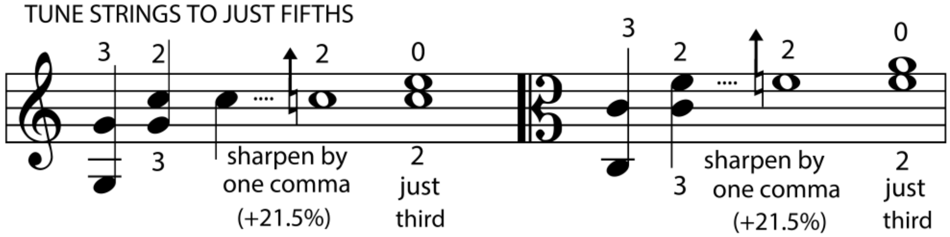
Constructing the Just Major Third

Once the just major sixth has been mastered, the just major third can be constructed using the first bar of this passage:



(4)

This major third will be at exact just consonance if the major sixth, the octave, and the open strings are all at exact just consonance. The second bar in (4) is a variation that challenges the student to play just major third, and then verify it by seeing if a just major sixth occurs when the third finger is released. The following confidence-building activity employs the concept of "hunting" for the just major third:



(5)

Here, the violin's initial C♮ will be too flat to make just consonance with the open string; it must be sharpened. If strings are tuned to just fifths, the upward "hunt" is one syntonic comma (21.5%). As discussed in the appendix, the initial C♮ must be sharpened by 16% if the strings are tuned to equal tempered fifths. Passage (5) should be practiced in three variations:

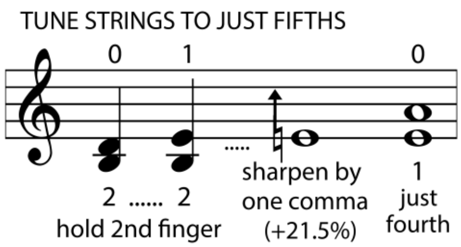
1. Hold second finger still and sharpen it after the double stop with the higher open string has begun. This is the simplest variation for students as it allows them to "hunt" for consonance.
2. Sharpen the second finger before playing the double stop. This teaches students to "learn the comma".
3. The most advanced exercise calls for playing the notes in reverse order in an attempt to test one's ear for the tempered major third (before hearing the just interval): On the violin, begin by tuning the C♮ to the slightly dissonant sound of a tempered major third against the open "E". Now hold the C♮ steady as the third finger G♮ is tuned to a perfect fourth. Only if the third finger G♮ is a consonant octave above the open "G", does the student know that the original tempered third was accurate.

Constructing the Just Minor Third

The following construction of the just minor third requires that the student can already play a just major third:

(6)

This minor third is exact (if the open strings and all other intervals are at exact just consonance.) The second bar in (6) serves to verify the students' ability to play a just minor sixth, and both bars can be practiced in reverse order. The confidence-building exercise involves the need to sharpen the E♮ in the following passage:

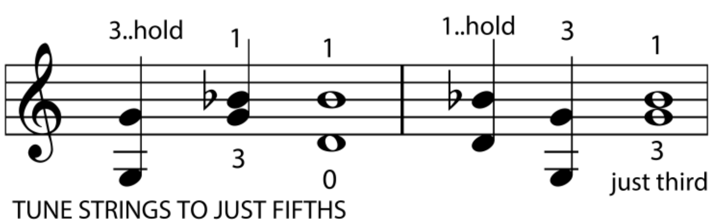


(7)

Here, the E♮ must be sharpened by 20% if the strings are tuned to tempered fifths. Repeat this passage in the three variations described after passage (5).

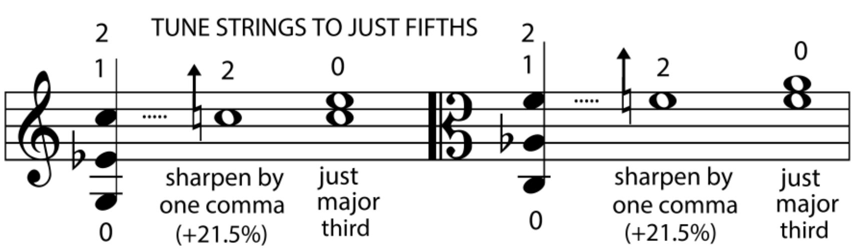
Constructing the Minor Sixth

Construct the minor sixth from an octave and minor third:



(8)

To build confidence, repeat the following in the three variations described after (5):

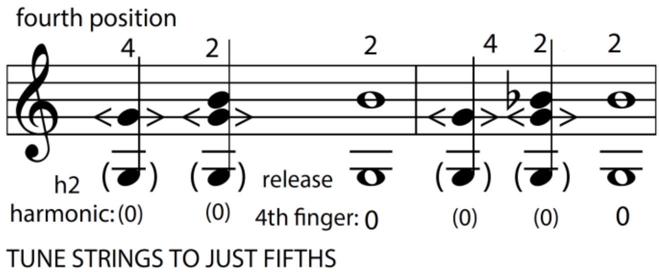


(9)

The adjustment is 16% if the open strings are equal tempered, as discussed in the appendix.

Constructing Just Intervals Larger than one Octave

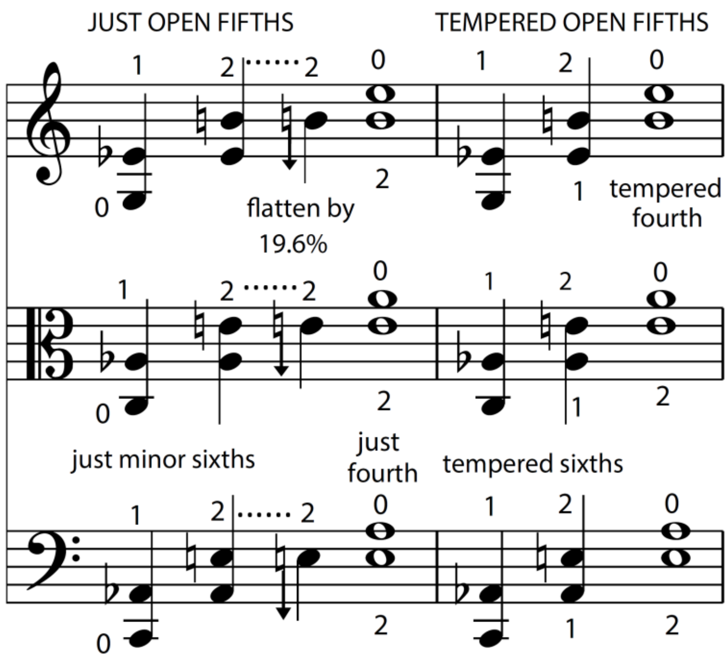
The fingerboard charts indicate two intonation tests that require compound intervals. They can be constructed by playing in fourth position with the 2nd finger and using the 4th finger to play the h2 harmonic on the "G" string, which converts the compound intervals into thirds:



(10)

Here the parenthesis (...) and brackets <…> describe a harmonic: Play the string indicated by parentheses, and the note actually sounded is shown in brackets.

The following passage explores the minor sixth:



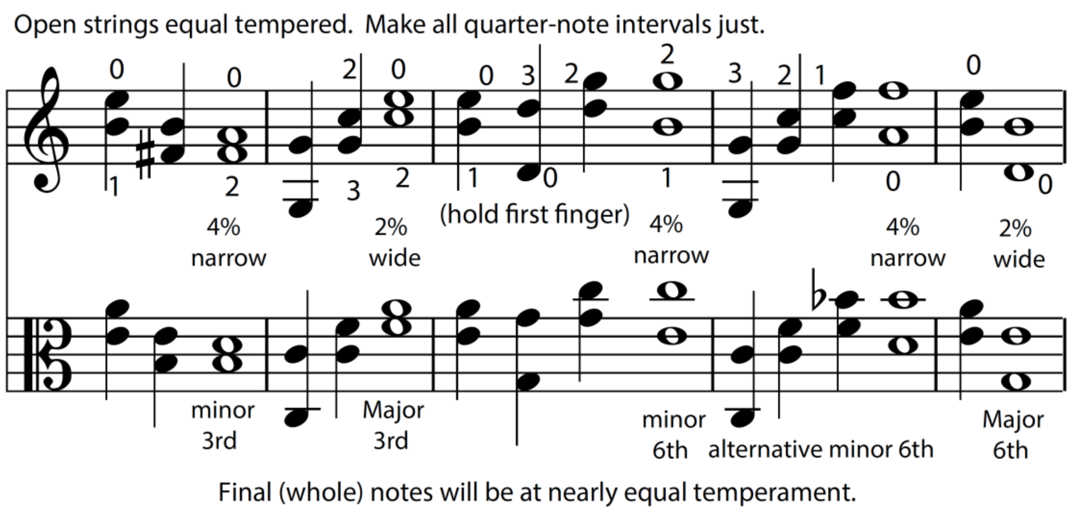
(11)

Repeat the first bar of (11) in the three variations discussed after (5). If the first bar of (11) is attempted with open strings tuned to the narrower tempered fifths, the violin must flatten the B♮

by 25.4% in order to make a just fourth with the open “E”. If the strings are equal tempered, the second bar in (11) can be attempted. If both minor sixths are equally dissonant, then both are at equal temperament. This temperament will be exact if the final perfect fourth is equal tempered (assuming equal tempered open strings).

Tempered Intervals

Here we demonstrate that equal tempered intervals can approximately constructed using just intervals. The instrument should be tuned to open tempered fifths. In the passages that follow, “quarter notes” indicate intervals played to just consonance. If the procedure is executed flawlessly, the resulting pair of “whole notes” will form an interval that is close to equal temperament. (For example in the first bar, the minor third will be 4% narrower than equal temperament.)



(12)

Issues in Psychology

One challenge string teachers face is that students might spend hours playing something incorrectly before an intervention can occur at the next lesson. The point of these activities is to enable students to self-correct while practicing alone. But the harmonic shifting exercises do not place fingers exactly where they belong. Little or no effort is made to teach students to distinguish between the different “comma-like” adjustments within the 14%-22% range. Passage (12) teaches students to play equal tempered intervals with errors of 2% or 4%. Will all this teach bad habits to our students? We offer no definitive answers, and therefore must be content with putting forth the issues involved.

Behavioral psychologist Arthur Staats (1996) introduced the concept of Basic Behavioral Repertoires (BBRs), which has been applied to sensory-motor skills by a number of researchers (Riedel, Heiby, & Kopetskie, 2001). The idea is that one must first master basic skills before progressing to more complex ones; much like learning how to print before progressing to cursive writing.

Studies of how people acquire a second or even third language might also be relevant. Perhaps basic skills such as shifting might be best learned as early as possible, just as language skills are best acquired at an early age. Anyone who has attempted to learn a third language will tell you that it was easier to become semi-fluent in the third language, than it was to achieve the same level in the second language. (Bialystok & Hakuta, 1999; Perani, *et al.*, 1998). This is especially true when one acquires the first language early on in development (Levelt, 1989).

Two other issues are a bit more problematical: First, if the tempered scale is analogous to learning a second language, then perhaps it needs to be learned first. Second, these interval tests do not work well in keys such as A or E major (since the scales are more “well-tempered” than “equal-tempered”.) The only way to hear the truly equal tempered scale is to spend long hours listening to a perfectly tuned keyboard instrument. The intonation tests based on just intervals are not being offered as a substitute for that. It is plausible, if not likely, that exposure to equal temperament should begin at the earliest possible stage in a child’s life.

Acknowledgements

We are grateful to Everet Kalcek for many helpful conversations about music instruction, as well as to David Smiley, a teacher and member of the San Francisco Symphony who introduced interval testing to young viola player back in the 1960s.

Appendix: Mathematica est diabolus in musica

All frequency ratios in the tempered scale are of the form, $2^{n/12}$, where n is an integer. Just intonation occurs when the ratio of frequencies is p/q, where p and q are reasonably small positive integers. [Plomp 1965] These fingerboard charts are based on four equations:

$$12 = 12\log_2 2 \ \text{(exactly)}$$

$$19 + \ 2\,\% \approx 12\log_2 3 \approx 19.0196 = 19 + 0.0196$$

$$28 + 14\% \approx 12\log_2 5 \approx 27.8631 = 28 - 0.1369 \approx 28 + 14\%$$

$$34 + 31\% \approx 12\log_2 7 \approx 33.6883 = 34 - 0.3117 \approx 34 + 31\%$$

The *syntonic comma* is $12\log_2\left(\frac{81}{80}\right) \approx 21.5\%$, and the *Pythagorean comma* which arises in passage (11) is $12\log_2\left(\frac{2048}{2025}\right) \approx 19.6\%$. The close proximity of logarithms involving {2, 3, 5} to whole numbers (to within 14%) helps explain why ratios involving these intervals are so suitable to the tempered scale. Logarithms are convenient because they allow us to represent ratios of frequencies as subtraction between logarithms. In fact, music students naturally think of “adding” the major third and minor sixth to make an octave, while mathemitican might view it as multiplication of frequency ratios: $(5/4)(8/5) = 2/1$. For example, consider the perfect fifth, or 7 halftones on the tempered scale. The frequency ratio, 3/2, is sharp by about 2%:

$$12\log_2\left(\frac{3}{2}\right) \approx 19.02 - 12.00 \ \ \approx 7.02$$

Performing this calculation with all the just intervals, we have:

Interval	Ratio	Half-tones	Error
octave	2/1	12	0%
fifth	3/2	$7.02 \approx 7 + 0.02$	+2%
fourth	4/3	$4.98 \approx 5 - 0.02$	−2%
major sixth	5/3	$8.84 \approx 9 - 0.16$	−16%
major third	5/4	$3.86 \approx 4 - 0.14$	−14%
minor third	6/5	$3.16 \approx 3 + 0.16$	+16%%
augmented fourth	7/5	$5.825 \approx 6 - .175$	−17.5%
minor sixth	8/5	$8.14 \approx 8 + 0.14$	+14%%

This table allows us to calculate the required pitch shifts in the passages involving construction of just intervals for instruments tuned to equal tempered open strings, although it takes a bit of practice to perform the calculations smoothly. Here it helps to view all just intervals as “wrong”: In passage (5), the initial C♮ is a just fourth, which is flat by 2%. But played against the open “E”, the final C♮ must be sharp by 14% (since major thirds are narrow). Hence the adjustment for (5) on tempered strings is 16%. In passage (7) the B♮ is flat by the just major third’s 16%, and the following pair of just fourths flatten the pitch by 2% each, so that the end result is a note that is 20% flat. In passage (9) the adjustment is 16%, since errors in the major and minor sixths partially cancel to yield a B♮ that is 2% flat, and needs to be 14% sharp to make a major third with the open “E”. In passage (11), the two just minor sixths combine to make a B♮ to sharp by 28%. But this B♮ needs to be 2% sharp to make a perfect fourth with open “E”, which reduces the required adjustment to approximately 26% (more precisely, $2 \times 13.69\% - 1.96\% \approx 25.4$.)

Since ratios involving smaller numbers tend to be more consonant, these intervals have been ranked according to the sum of the numerator plus denominator: For example, 2+1 = 3 is smaller than 3+2 =5, so we rank the octave as more consonant than the perfect fifth. This ranking roughly matches the ranking of consonance most people perceive. Malmberg (1918) used a jury of musicians and psychologists to rank the consonance in roughly the same order, except that the minor sixth is regarded as being considerably more consonant: 2/1, 3/2, 5/3, 4/3, 5/4, 8/5, 6/5, 7/5. .

The Pythagorean intervals are approximations to the tempered scale that use higher order fractions that do not contain the integer {5}

Pythagorean	Ratio	Half-tones	Error
minor third	32/27	2.94 =3 -0.06	-6%
major third	81/64	4.08 =4+0.08	8%
minor sixth	128/81	7.92=8-0.08	-8%
major sixth	27/16	9.06=9+0.06	6%

Intervals containing ratios with {7} have been called “the devil in music” (Barbieri, 1991). The 31% error associated with this number makes it closer to the midpoint between two halftones than it is to either 33 or 34. Nevertheless, something close to a tempered interval can be achieved by arranging the error in {5} to partially offset the error in {7}:

$$(34 - .312) - (28 - .137) = 6 - .175$$

The adjacent string test associated with this interval is shown in the fingerboard charts, but not highlighted because the test is neither close to an equal tempered interval, nor very consonant.

Though the 7/5 interval is not well suited as an intonation test, the attempt to hear this resonance will interest those who love mathematics: Since the tritone’s tempered frequency ratio, $2^{6/12}$, is the square root of two, and since 49 almost equals 50, the following three numbers are nearly equal:

$$\frac{7}{5} < \sqrt{2} < \frac{10}{7}$$

We see that the fingerboard charts require additional thin blue circles that are 17% flatter than the "G" string's tempered G#. Students' attempt hear these just intervals as consonant might prove futile, perhaps because there are other pairs of number that approximate the square root of two, leading to a cluster of just intervals: $25/18 < 7/5 < 45/32 < \sqrt{2} < 64/45 < 10/7 < 36/25$. Mathematically inclined students will appreciate that this symmetric pairing of all just intervals near the triton is associated with a well-known algebraic identity: $\sqrt{2} = 2/\sqrt{2}$.

References

Barbieri, Patrizio, and Sandra Mangsen. "Violin intonation: a historical survey."Early music 19, no. 1 (1991): 69-88.

Barbieri, Patrizio. *Enharmonic: instruments and music 1470-1900; revised and translated studies*. Vol. 2. Il Levante, 2008.

Bialystok, Ellen, and Kenji Hakuta. *In other words: The science and psychology of second-language acquisition*. Basic Books, 1994.

Burns, Edward M., and W. Dixon Ward. "Intervals, scales, and tuning." The psychology of music 2 (1999): 215-264.

Loosen, Franz. "Intonation of solo violin performance with reference to equally tempered, Pythagorean, and just intonations." The Journal of the Acoustical Society of America 93 (1993): 525.

Malmberg, Constantine Frithiof. "The perception of consonance and dissonance." Psychological Monographs: General and Applied 25.2 (1918): 93-133.

Perani, Daniela, Eraldo Paulesu, Nuria Sebastian Galles, Emmanuel Dupoux, Stanislas Dehaene, Valentino Bettinardi, Stefano F. Cappa, Ferruccio Fazio, and Jacques Mehler. "The bilingual brain. Proficiency and age of acquisition of the second language." *Brain* 121, no. 10 (1998): 1841-1852.

Plomp, Reinier, and Willem JM Levelt. "Tonal consonance and critical bandwidth." *The journal of the Acoustical Society of America* 38 (1965): 548.

Riedel, Helmut PR, Elaine M. Heiby, and Stephen Kopetskie. "Psychological behaviorism theory of bipolar disorder." *The Psychological Record* 51, no. 4 (2011): 1.

Ross, Barry. *A violinist's guide for exquisite intonation*. Alfred Publishing Co., Inc., 1989.

Sassmannshaus, K. (n.d.). Intonation. *The Sassmannshaus Tradition for Violin Playing.* Retrieved December 6, 2012, from http://www.violinmasterclass.com/en/masterclasses/intonation

Staats, Walter W., and Arthur W. Staats. *Behavior and personality: Psychological behaviorism*. Springer Publishing Company, 1996.

Sundberg, J. E. F., & Lindqvist, J. (1973). Musical octaves and pitch. The Journal of the Acoustical Society of America, 54, 922.

Vandegrift, G. (1994, Fall). Why Bach Sounds Funny on the Piano. American String Teacher, 44(4), 12-18.

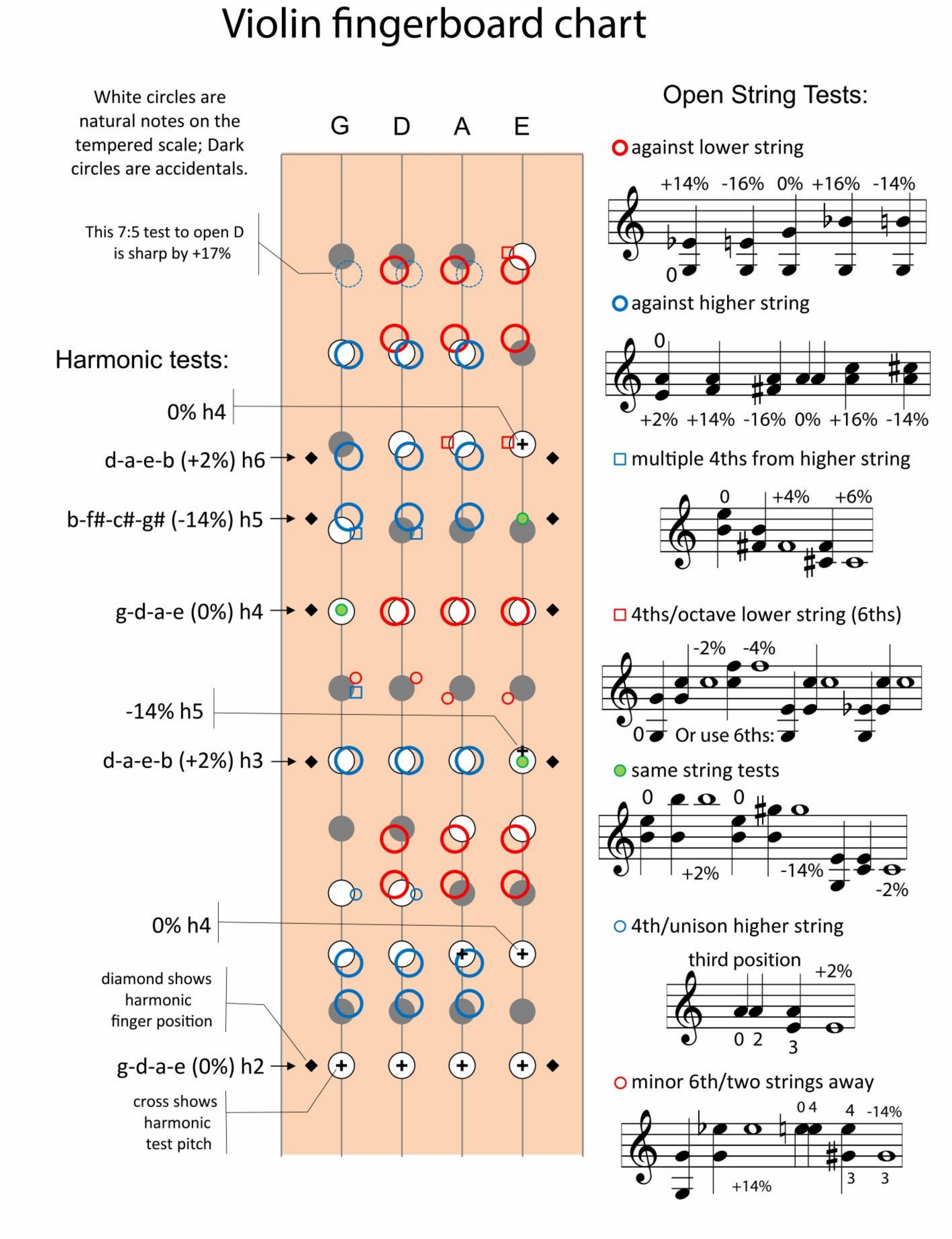

Violin fingerboard chart
White circles are natural notes on the tempered scale; Dark circles are accidentals.
G D A E
This 7:5 test to open D is sharp by +17%
Harmonic tests:
0% h4
d-a-e-b (+2%) h6
b-f#-c#-g# (-14%) h5
g-d-a-e (0%) h4
-14% h5
d-a-e-b (+2%) h3
0% h4
diamond shows harmonic finger position
g-d-a-e (0%) h2
cross shows harmonic test pitch
Open String Tests:
against lower string
+14% -16% 0% +16% -14%
against higher string
+2% +14% -16% 0% +16% -14%
multiple 4ths from higher string
+4% +6%
4ths/octave lower string (6ths)
-2% -4%
Or use 6ths:
same string tests
+2% -14% -2%
4th/unison higher string
third position
+2%
minor 6th/two strings away
+14% -14%

Figure 1: Violin fingerboard chart

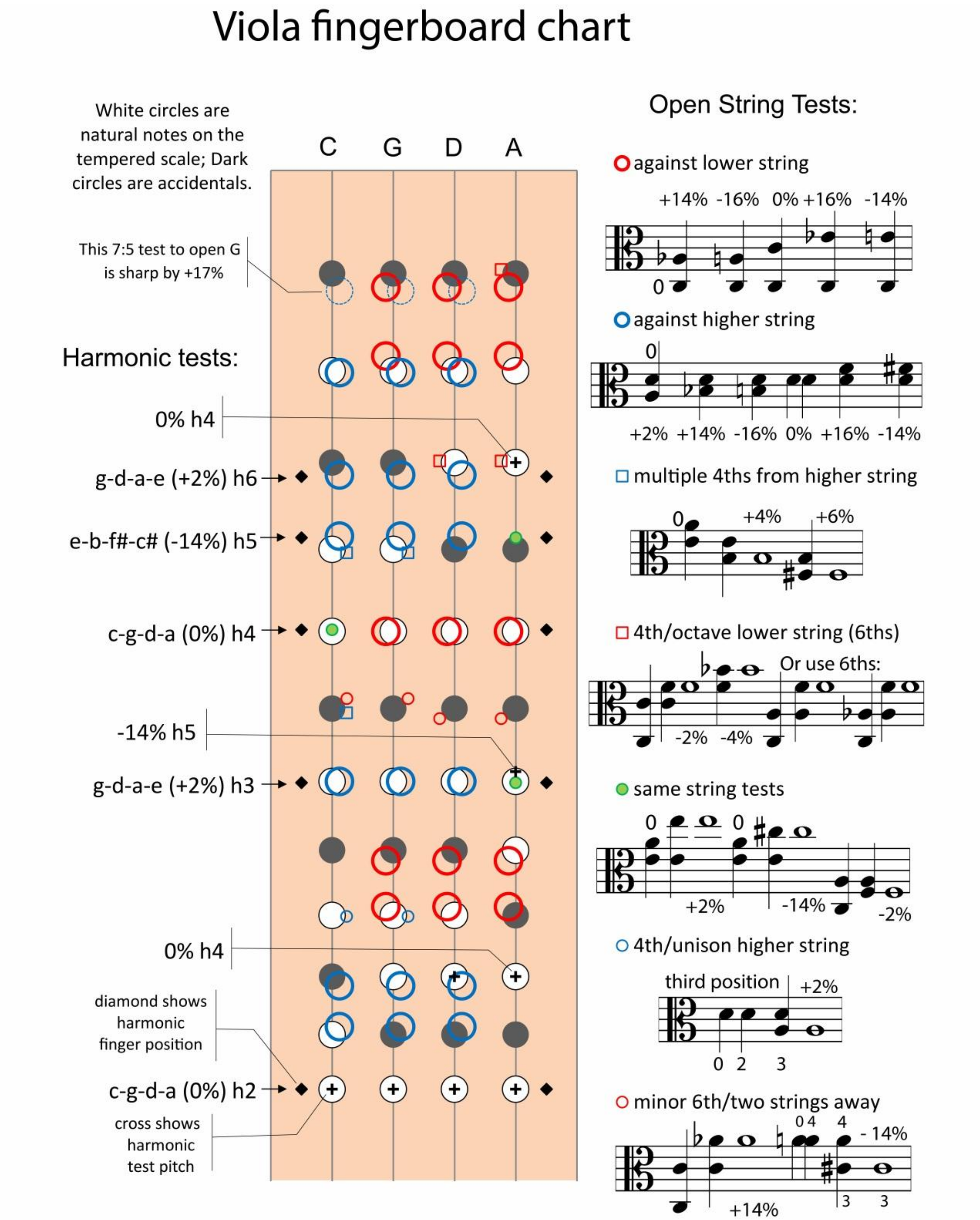


Figure 2: Viola fingerboard chart

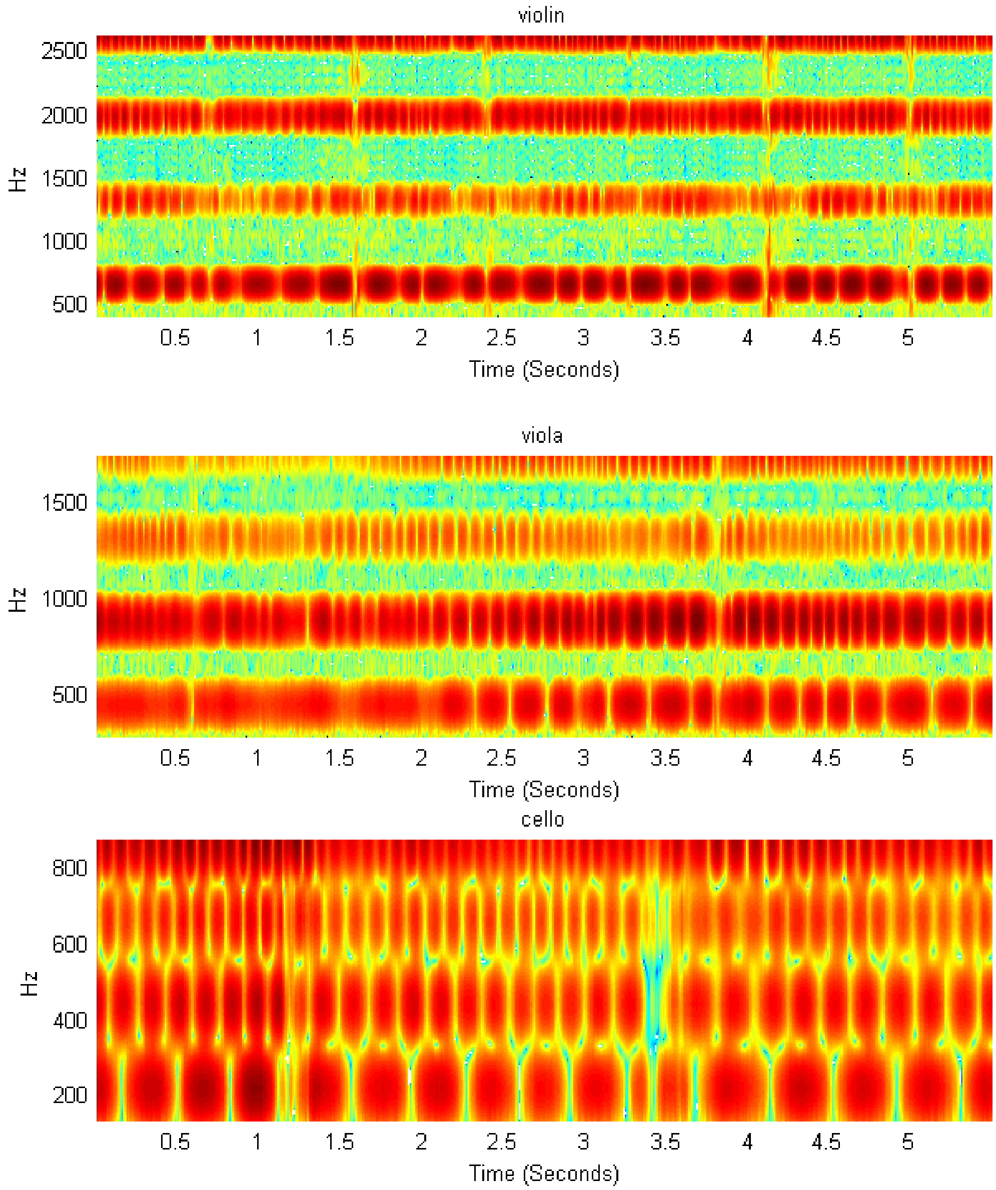


Figure 3: Spectrograms for violin, viola, and cello. A double stop is played between the highest string and a fingered note on the second highest string. The fingered note is an octave and just major sixth above the lowest string. Changes in the direction of the bow are responsible for sudden disturbances (see, for example the violin's signal just beyond 1.5 seconds, or the cello at 3.5 seconds.)

Remove from submitted document 266d, 266e, 266f. Alt-X: xx ♭ xx♮xx♯x 10/30/2013 4:03 PM