Stefan–Boltzmann law

Introduction:

This is an example of the so-called "back-of-the-envelope" calculation that physics majors are trained to perform. This one could be grasped by most 2nd year college physics majors. Skip it if you wish. I did it because I teach that Mars has much less greenhouse warming than Earth, and I just wanted to see so for myself.

Excerpts taken from Wikipedia, the free encyclopedia

The Stefan–Boltzmann law, also known as Stefan's law, states that the total energy radiated per unit surface area of a black body in unit time (known variously as the black-body irradiance, energy flux density, radiant flux, or the emissive power), j*, is directly proportional to the fourth power of the black body's thermodynamic temperature T (also called absolute temperature):

$$j^{\star} = \sigma T^4.$$

The irradiance j* has dimensions of energy flux (energy per time per area), and the SI units of measure are joules per second per square metre, or equivalently, watts per square metre. The SI unit for absolute temperature T is the kelvin.

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To find the total absolute power of energy radiated for an object we have to take into account the surface area, A(in m2):

$$P = Aj^{\star} = A\varepsilon\sigma T^4.$$

The constant of proportionality σ, called the Stefan–Boltzmann constant or Stefan's constant, is non-fundamental in the sense that it derives from other known constants of nature. The value of the constant is

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.670400 \times 10^{-8}\mathrm{J\,s^{-1}m^{-2}K^{-4}}$$

where k is the Boltzmann constant, h is Planck's constant, and c is the speed of light in a vacuum. Thus at 100 K the energy flux density is 5.67 W/m2, at 1000 K 56,700 W/m2, etc. Here another constant is included to account for imperfect or "gray" blackbodies. The constant, ε<1 for such gray bodies, but we shall set ε = 1 and make the simplifying assumption that planets are perfect black bodies.

The law is valid only for ideal black objects, the perfect radiators, called black bodies.

Temperature of the Earth

We can calculate the effective temperature of the Earth TE by equating the energy received from the Sun and the energy transmitted by the Earth, under the black-body approximation:

$$
\begin{aligned}
T_E &= T_S\sqrt{\frac{r_S}{2a_0}} \\
&= 5780\ \mathrm{K} \times \sqrt{\frac{696 \times 10^6\ \mathrm{m}}{2 \times 149.598 \times 10^9\ \mathrm{m}}} \\
&\approx 279\ \mathrm{K}\ ,
\end{aligned}
$$

where T_S is the temperature of the Sun, r_S the radius of the Sun, and a_0 is the distance between the Earth and the Sun. Thus resulting in an effective temperature of 6°C on the surface of the Earth.

The above derivation is a rough approximation only, as it assumes the Earth is a perfect blackbody. The same equilibrium planetary temperature would result if the planet's emissivity and absorptivity were reduced by some constant fraction at all wavelengths, since the incoming and outgoing powers would still match at the same temperature (this equilibrium temperature would no longer fit the definition of effective temperature, however).

The real Earth does not have this "grey-body" property. The terrestrial albedo is such that about 30% of incident solar radiation is reflected back into space; taking the reduced energy from the sun into account and computing the temperature of a black-body radiator that would emit that much energy back into space yields an "effective temperature", consistent with the definition of that concept, of about 255 K. However, compared to the 30% reflection of the Sun's energy, a much larger fraction of long-wave radiation from the surface of the earth is absorbed or reflected in the atmosphere instead of being radiated away, by greenhouse gases, namely water vapor, carbon dioxide and methane. Since the emissivity (weighted more in the longer wavelengths where the Earth radiates) is reduced more than than the absorptivity (weighted more in the shorter wavelengths of the Sun's radiation), the equilibrium temperature is higher than the simple black-body calculation estimates, not lower. As a result, the Earth's actual average surface temperature is about 288 K, rather than 279 K.

If we apply the same (highly idealized) model of a perfect black body to Mars, and use the fact that Mars is 1.52 au from the Sun to calculate the effective temperature of Mars to be,

$$T_M \approx \sqrt{\frac{1}{1.52}}\ 279\ K \approx 226\ K$$

From various sources on the internet, I estimate the actual surface temperature of Mars to be 216±5 K . Hence:

The actual Earth-Mars temperature difference is 288 K – 216 K = 72 ± 5 K

The simple blackbody calculation yields a difference of 279 K – 226 K = 53 K

The greenhouse effect is smaller on Mars, just as my teaching materials stated!