Mathematical Models of Diffusion within a Bone Bed

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Abstract

MATHEMATICAL MODELS OF DIFFUSION WITHIN A BONE BED

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The issue of diffusion often enters the analysis of paleoecological abundance and diversity data from bone beds containing dermal and dental elements from amphibians, chondrichthyans, acanthodians, placoderms, or osteichthyans. We are in the initial stages of collecting various mathematical models for this diffusion, using various models from physics for inspiration. The simplest diffusion model is the classical theory of Einstein and Smoluchowski, used to analyze Brownian motion. Variations of this classical theory include anisotropic diffusion coefficients (a tensor theory), as well as inhomogeniety in the diffusion and/or drift velocity. Studies of particle diffusion in thermonuclear plasma confinement devices have led to models of far greater complexity. Each level of complexity adds more unknown parameters, and hence a drastic decrease in the prospects of a model that can be tested by analyzing samples taken from the bonebed. Our search is therefore for a bonebed with the following properties:

1. The mathematical model must contain only a few unknown parameters.

2. The bonebed should be either sufficiently large, or be associated with a sufficiently large number on nearly identical bonebeds. This will permit inferences from a sample to be tested on larger sample for statistical significance.

3. There should be maximum understanding and minimal controversy concerning both the original ecosystem, as well as the subsequent geological processes associated with the bonebed.

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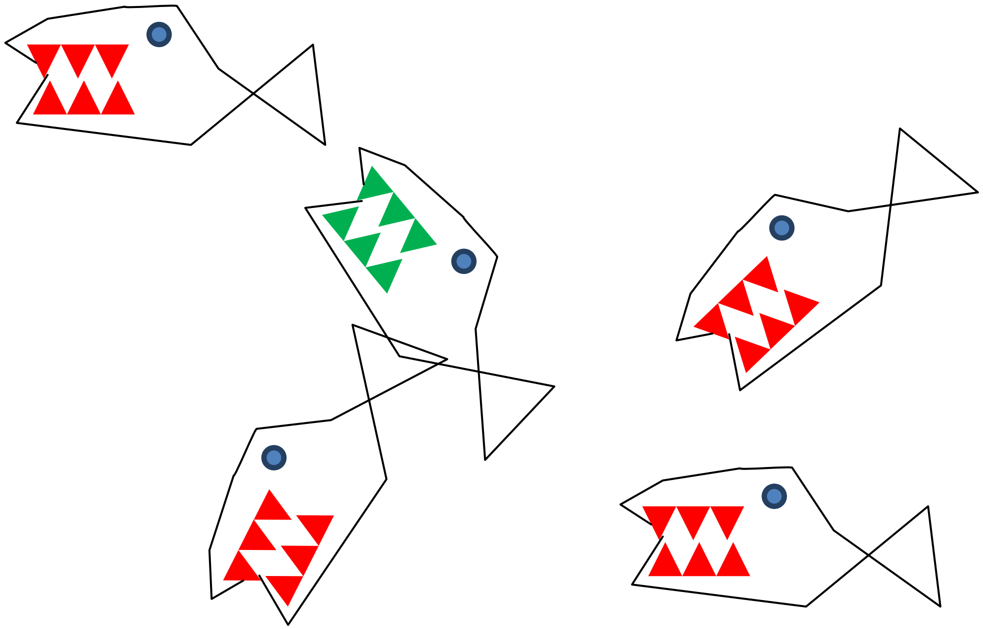
Statement of problem

- A shark tooth is found in a bone bed.
- What is the probability a nearby tooth of the same species came from the same individual?

Five identical fish die – one destined to be "discovered":



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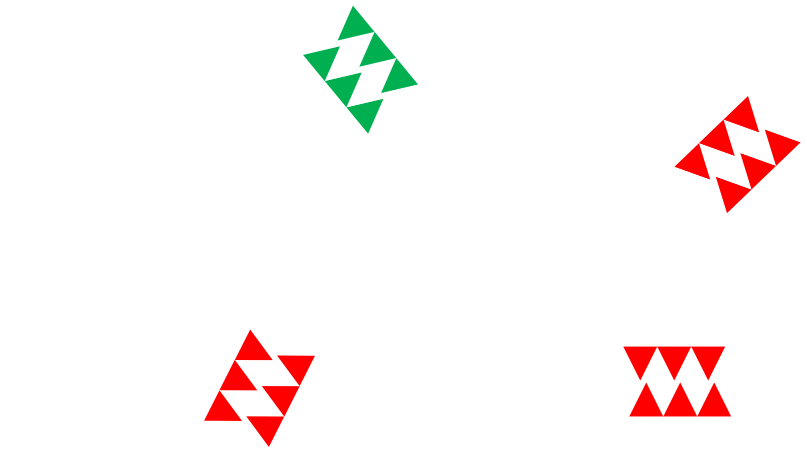
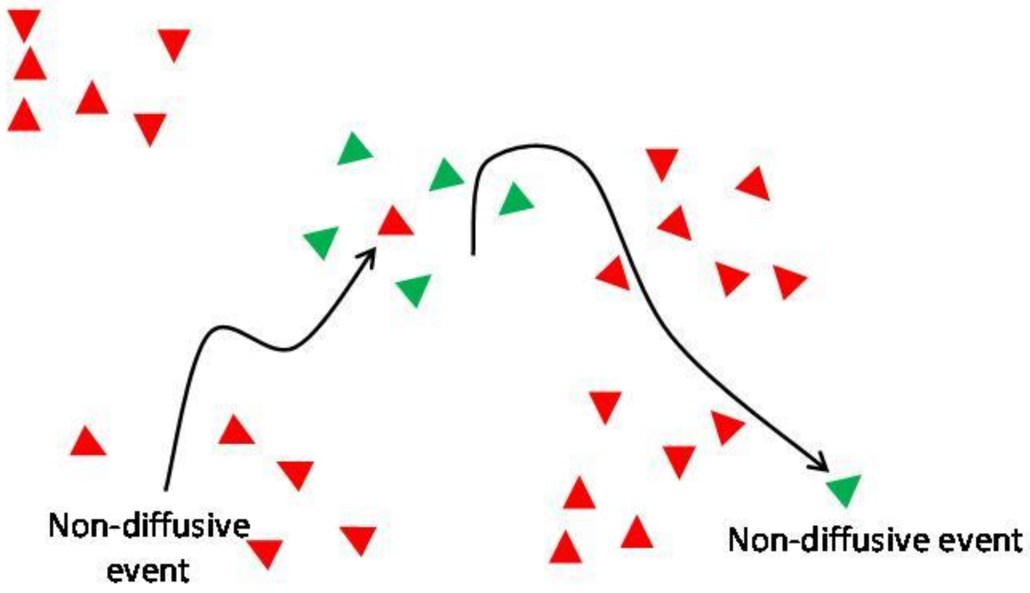
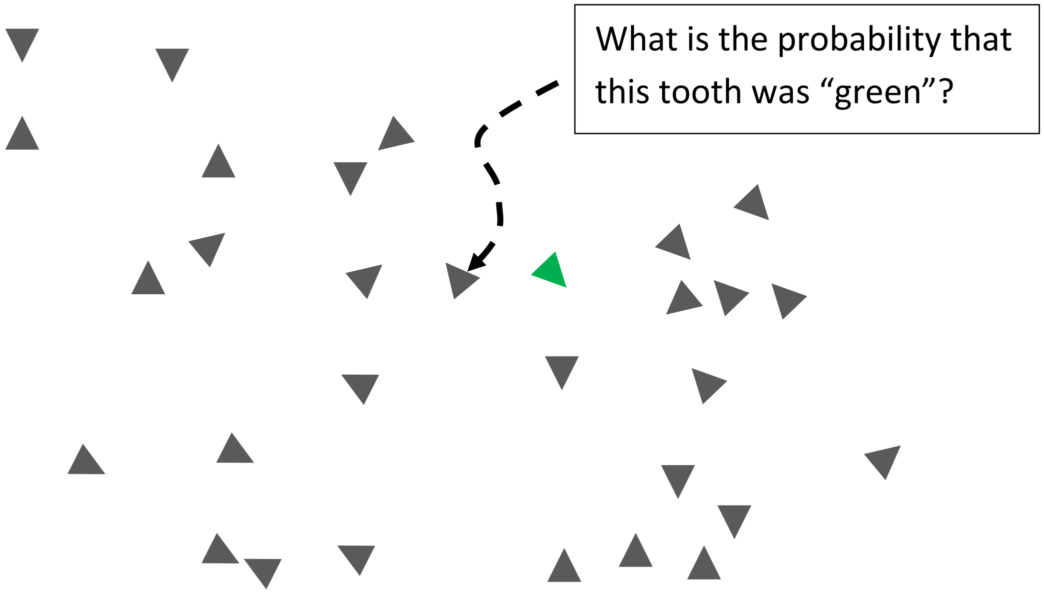


The teeth diffuse and also may experience non-diffusive events



When the tooth is finally discovered…



What kind of problem is this?

Easy

- Bayesian Probability
- Integral-Differential Diffusion Equation.
  - Easy to solve numerically
  - Solutions easy to implement (due to linearity)
  - **Solution often "guessed" w/o computer**

Difficult

- Too many unknown parameters
  - Unlike Brownian motion: No "thermal equilibrium"
  - Efforts to "force" an analogy to thermodynamics likely to fail (a lesson from Plasma Physics)
  - **Horrible experimental conditions (no lab!)**

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To me this seems like an intractable problem!

Fortunately we make this problem much easier:

- Let P be the probability that the second tooth came from the same individual.

- If we can establish $P < 0.1$, then we can state:

$$P = 0$$

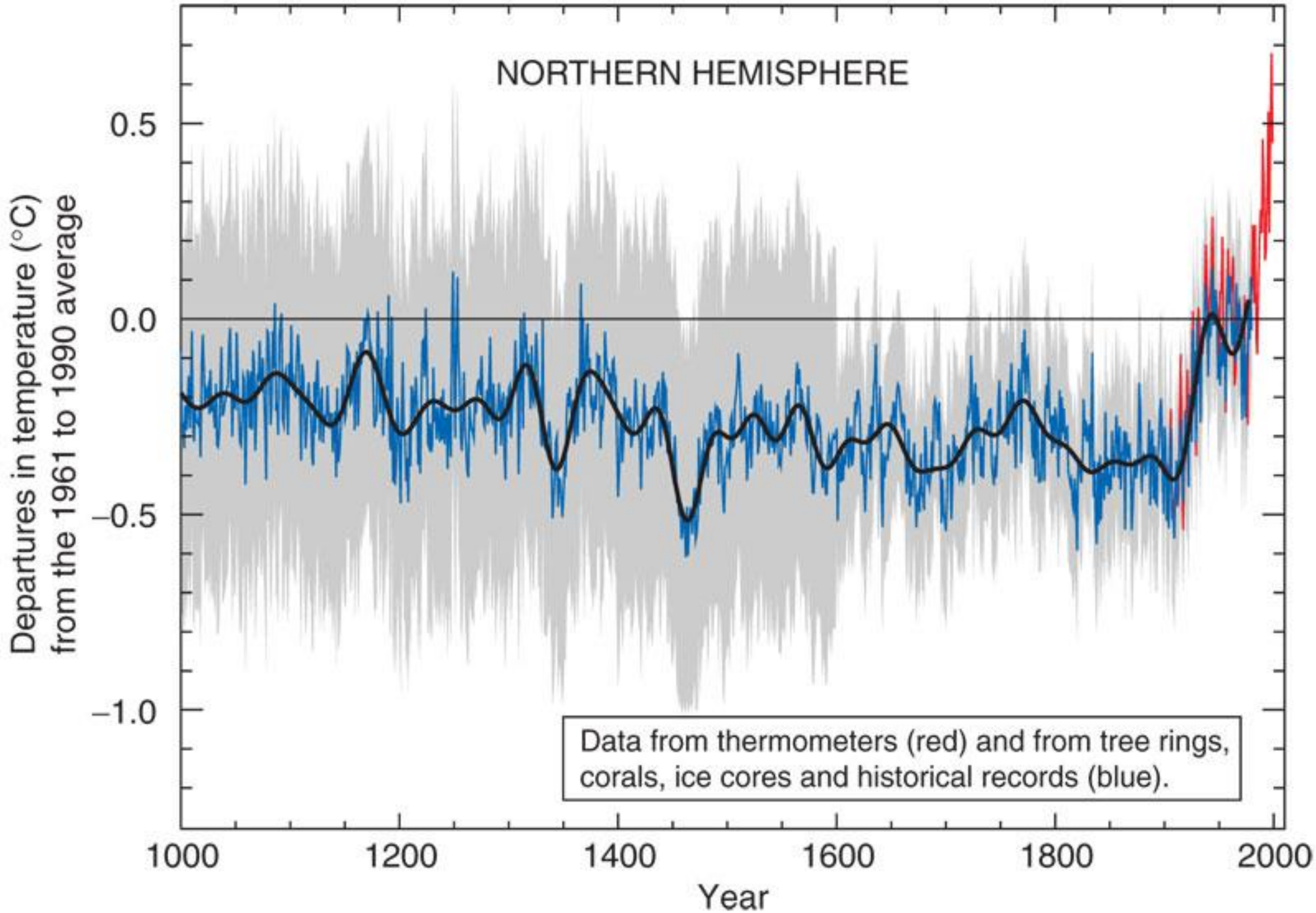
with 90% confidence.

Important caveat

Science based on a flawed model will lead

to an incorrect confidence level.

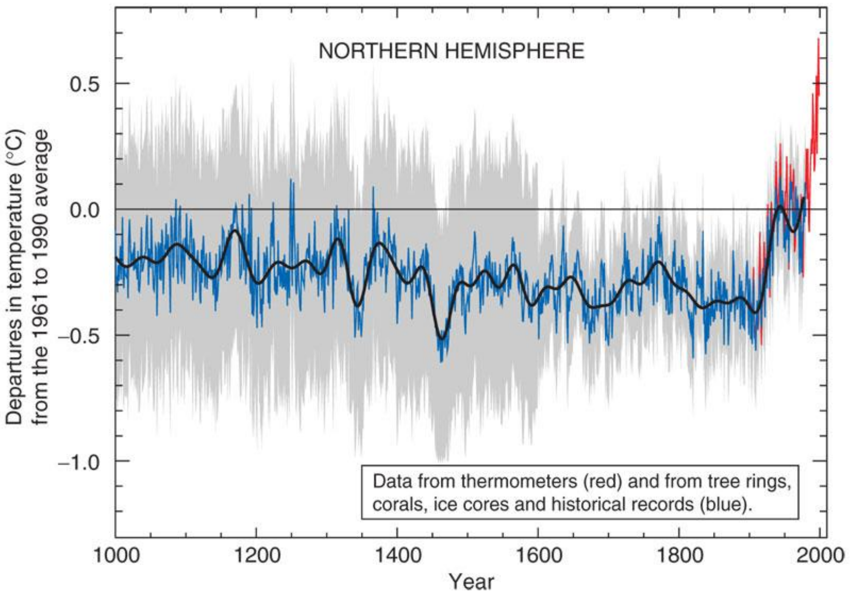
Consider an iconic example:



Does this graph suggest that we know the Northern Hemisphere's average temperature…..

- 1000 years ago
- plus or minus 0.4 degrees,
- with 95% uncertainty

?



What is statistical confidence?

- I think the graph implies that more data from
  - Coral reefs
  - Tree rings
  - Ice cores

would not change the results by more than 0.4°C

What is statistical confidence?

- Whether tree rings can actually determine past temperatures is not a question of

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What is statistical confidence?

- Whether tree rings can actually determine past temperatures is not a question of

  mathematical statistics,

  but a question of biology

The conundrum of scientific uncertainty

- *The procedure for declaring uncertainty must be made in a completely objective way.*

- *Only mathematical formulas can yield such objectivity.*

- *The only available mathematical model is that proposed by the author(s).*

By its very nature, the calculation of a Confidence Interval requires circular reasoning.

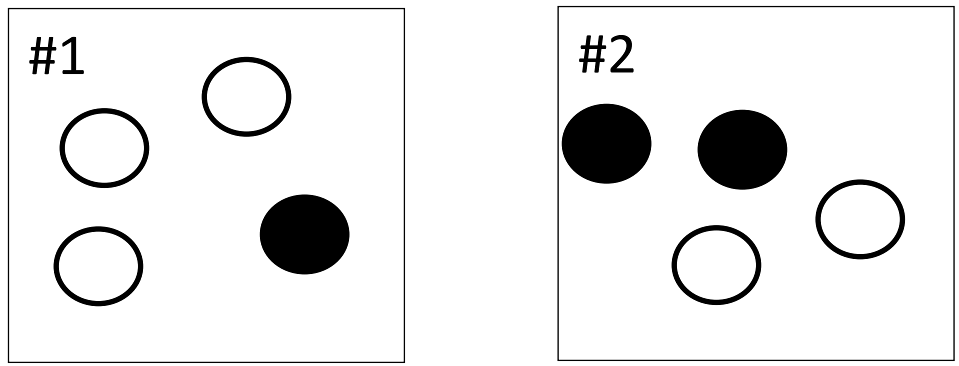
Two loose ends concerning the "easy" part of this problem:

- What is Bayesian Statistics?
  - "if/then" statistics where new info changes the odds

- Why is it easy to construct a model of diffusion.
  - The system is LINEAR (to excellent approximation), which allows a simple "Green's Function" solution of the form:

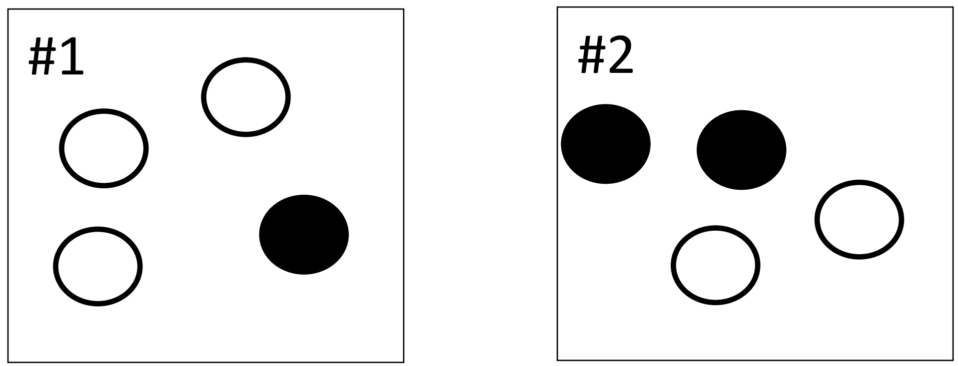
Where might a single bone go?

Bayesian Statistics



- A box is randomly selected and a marble is randomly selected from that box.
- What is the probability that the marble came from box #1?

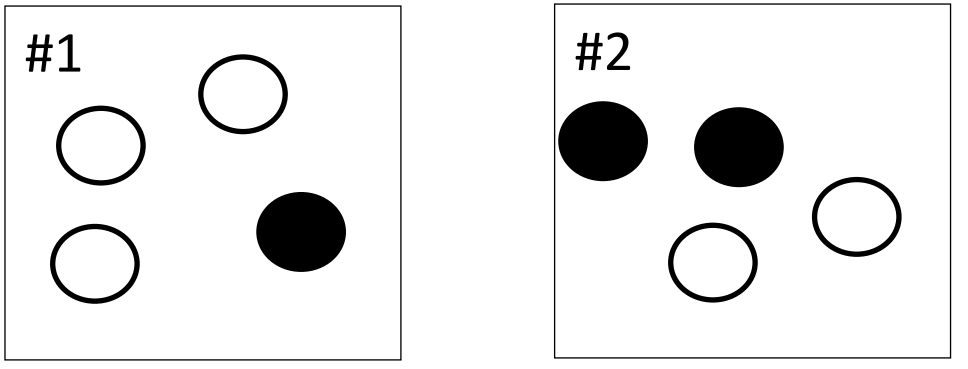
Bayesian Statistics



Since we have no reason to assume that one box is more “likely” than the other, our *a priori* answer is:

$$P(\#1) = 50\% = 0.5$$

Bayesian Statistics



If we discover that the marble selected was white, then P(#1) grows:

$$P(H_1|E) = \frac{P(E|H_1)\,P(H_1)}{P(E|H_1)\,P(H_1)\ +\ P(E|H_2)\,P(H_2)}$$

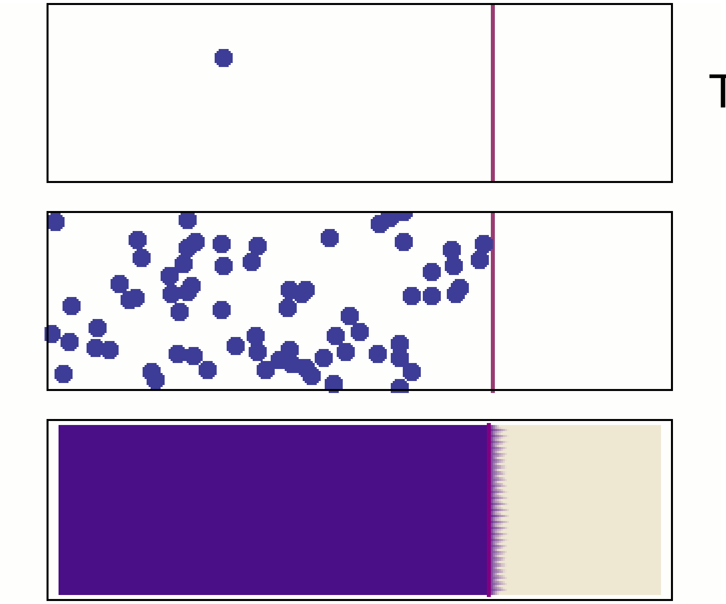
$$= \frac{0.75 \times 0.5}{0.75 \times 0.5 + 0.5 \times 0.5}$$

$$= 0.6$$

Guessing the diffusion model

- Consider just one bone
- Pick a confidence interval (e.g. 68%)
- Sketch where you think the bone might go as "time marches on"

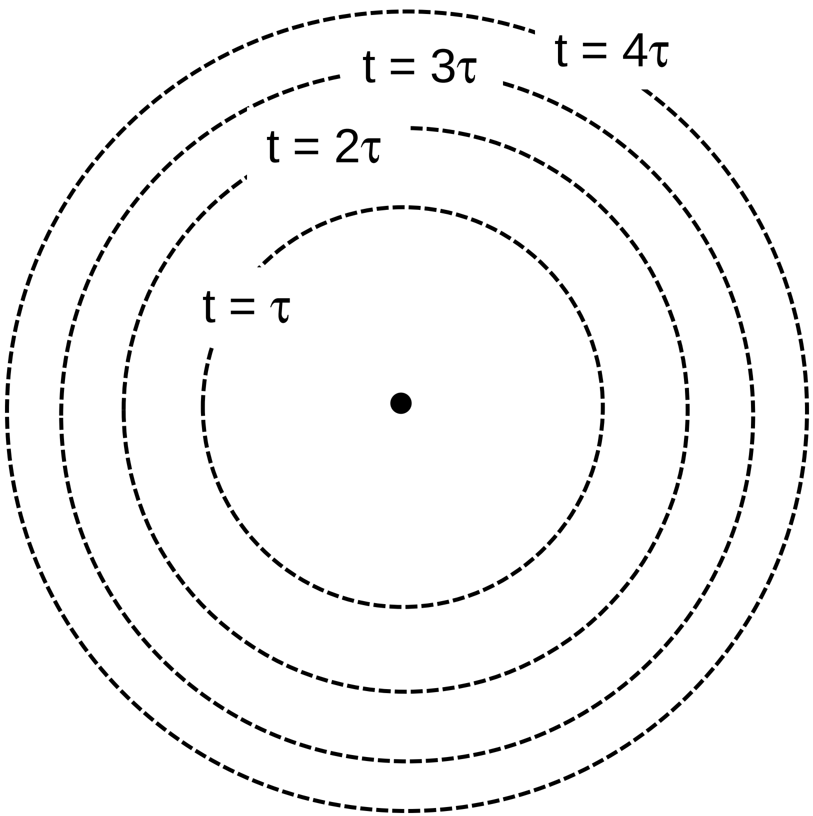
Diffusion can usually be considered one particle at a time.



Animation courtesy of Wikipedia [Fick's Law of Diffusion]

Guessing the diffusion model

"Thermal" (Brownian) Motion



Radius =R(t)

R(t)= $(2Dt)^{1/2}$

D = diffusion coef.

D = λ^2/τ

τ = "step time"

λ= "step length"

Guessing the diffusion model

- It is unlikely that we can calculate the geological diffusion coefficient, or that the concept even exists for non-thermal processes.
- But we might be able to estimate typical probability distributions if we had the right bone bed.

Guessing the diffusion model

Hypothetical "educated guess"

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