

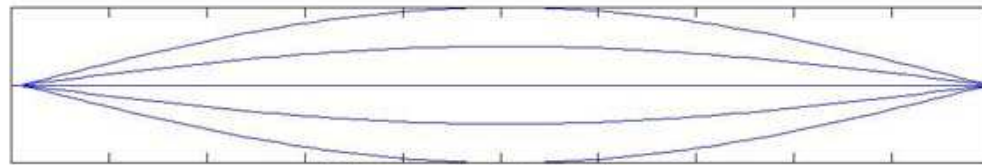
What is musical harmony*?

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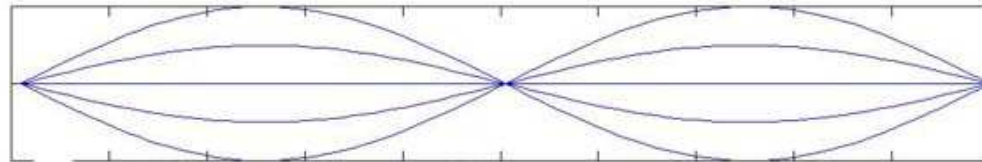
* Musicians refer to today's topic as *consonance*.

Normal Modes* of a String



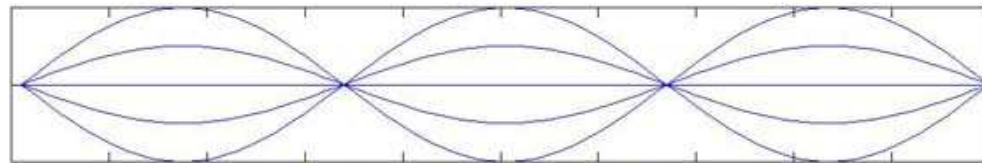
Node spacing L , frequency f_1

Mode 1



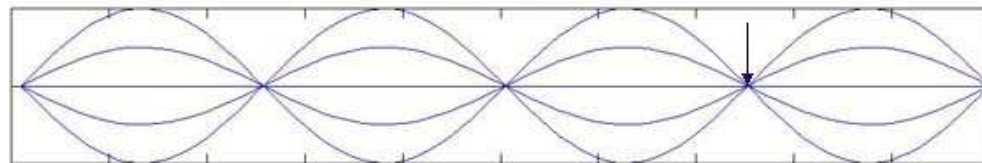
Node spacing $L/2$, frequency $2 f_1$

Mode 2



Node spacing $L/3$, frequency $3 f_1$

Mode 3



Node spacing $L/4$, frequency $4 f_1$

Mode 4

*Musicians call them *Harmonics*

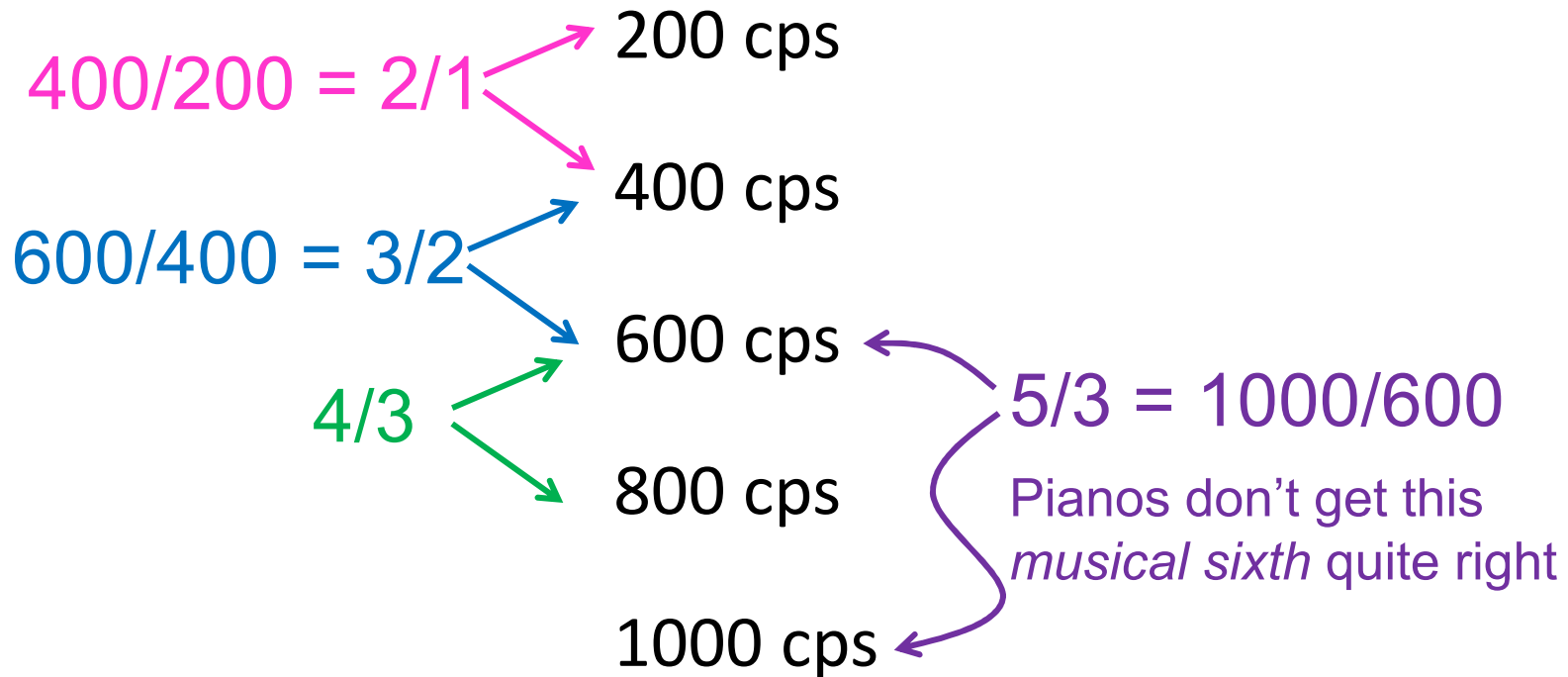
Typical measurements on a snaky spring

Count cycles for 10 seconds

- 11 cycles for mode 1
- 22* cycles for mode 2
- 33* cycles for mode 3

*It is common to be off by one cycle: counting 34 cycles instead of 33 cycles for mode 3, for example

Cycles per second for viola*

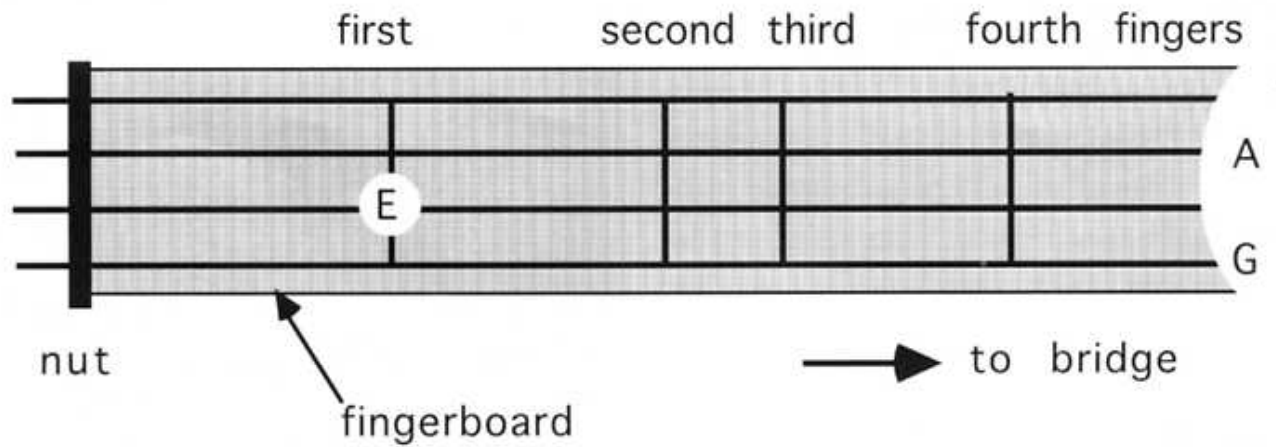


*Rounded for convenience.

Musical Terms for the Resonances

Minor third	$6/5$
Major third	$5/4$
Fourth	$4/3$
Fifth	$3/2$
Sixth	$5/3$
Octave	$2/1$

Two ways to get from G to E



Up one Musical Sixth

$$\frac{5}{3}$$

Up two Fifths and Down a Fourth

$$\frac{3}{2} \times \frac{3}{2} \times \frac{3}{4}$$

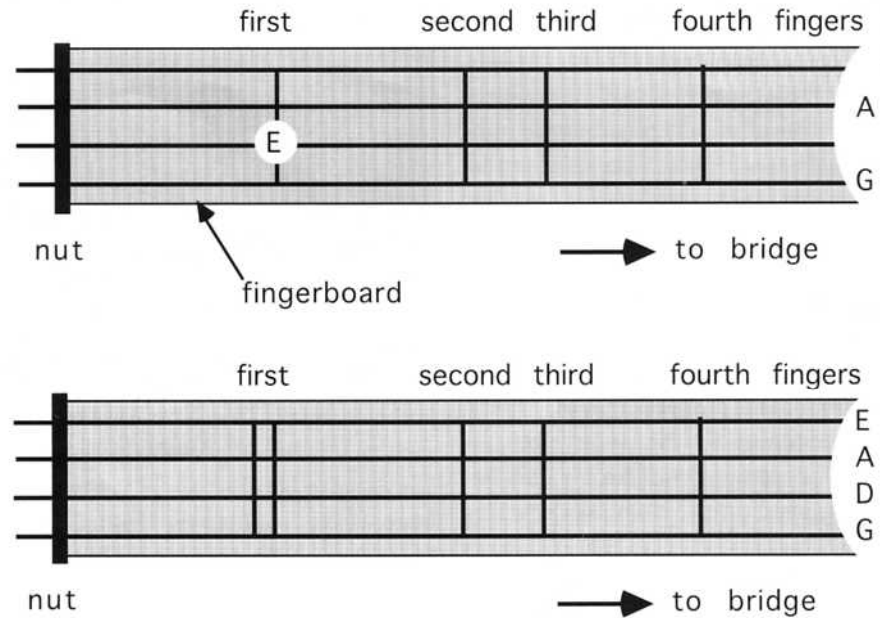
Cross-multiply the fraction

$$\frac{5}{3} = \left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right) \cdot \left(\frac{3}{4}\right) \quad ?$$

$$(5 \cdot 2)(2 \cdot 4) = (3 \cdot 3) \cdot (3 \cdot 3) \quad ?$$

$$80 = 81 \quad ?$$

What's the difference?



- The two lines for the “first” finger show the two choices for the note E.

either way is
Good Enough for Jazz!



And for Bach *



* Most of the time...
The “tempered” scale is
based on compromise.

Ways to obtain the ratio (f_E/f_G)

$$\frac{5}{3} = 1.6666 \dots$$

$$\frac{27}{16} = 1.6875000 \dots$$

$$\frac{5/3}{27/16} = 1.013$$

off by 1.3%

The tempered scale:

$$2^{9/12} = 1.681792831 \dots$$

$$\frac{5/3}{2^{9/12}} = 1.009$$

off by 0.9%