

Optimization: Logistics Network Configuration

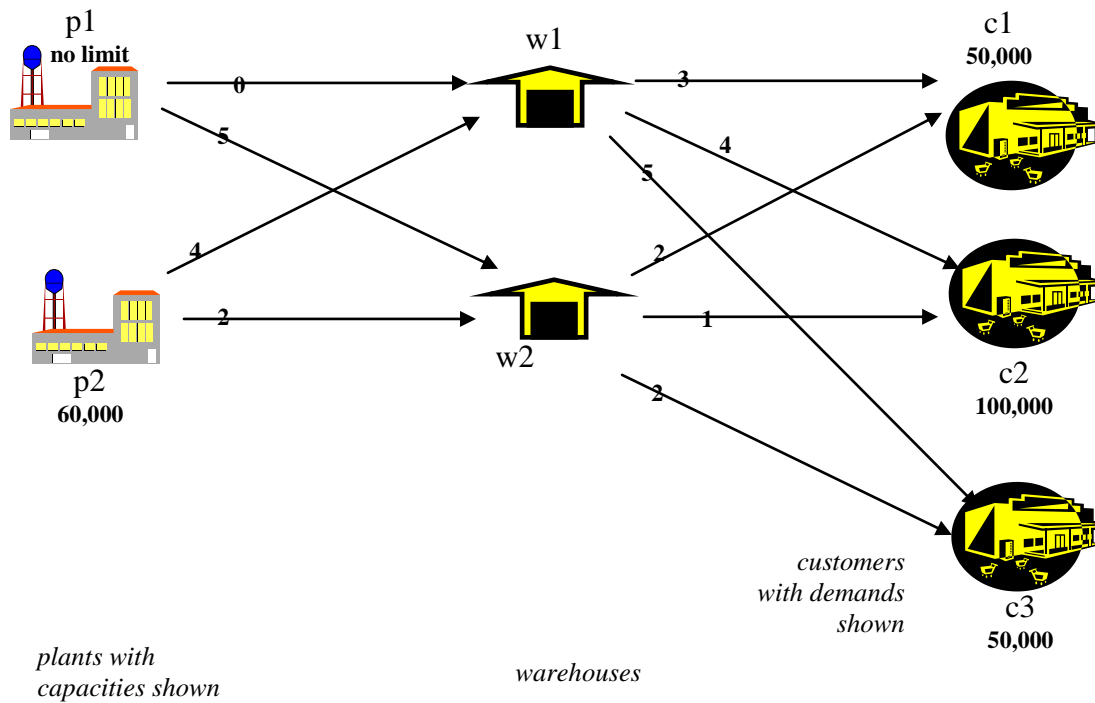


Figure 1. A logistics network; unit transportation costs are given on each connecting link. [Example 3-2 from *Designing and Managing the Supply Chain* by D. Simchi-Levi et al]

Goal: Find a plan of distribution to minimize total transportation cost, while satisfying the demand at each customer location and the plant capacity limitations.

The problem of determining the optimal logistics distribution in Figure 1 is an example of what is known as a linear program, and specifically in this context the “transshipment problem.” We have the following decision variables:

- $x(p1,w1)$, $x(p1,w2)$, $x(p2,w1)$ and $x(p2,w2)$ will be the flows from the plants to the warehouses;
- $x(w1,c1)$, $x(w1,c2)$, $x(w1,c3)$ will be the flows from the warehouse w1 to customer zones c1, c2 and c3.
- $x(w2,c1)$, $x(w2,c2)$, $x(w2,c3)$ will be the flows from warehouse w2 to customer zones c1, c2 and c3

The problem we want to solve is:

$$\text{minimize } 0x(p1,w1) + 5x(p1,w2) + 4x(p2,w1) + 2x(p2,w2) + 3x(w1,c1) + 4x(w1,c2) + 5x(w1,c3) + 2x(w2,c1) + 1x(w2,c2) + 2x(w2,c3)$$

subject to the following constraints:

| | |
|--|---------------------------------|
| $x(p2,w1) + x(p2,w2) \leq 60000$ | <i>capacity, plant 2</i> |
| $x(p1,w1) + x(p2,w1) = x(w1,c1) + x(w1,c2) + x(w1,c3)$ | <i>conservation of flow, w1</i> |
| $x(p1,w2) + x(p2,w2) = x(w2,c1) + x(w2,c2) + x(w2,c3)$ | <i>conservation of flow, w2</i> |
| $x(w1,c1) + x(w2,c1) = 50000$ | <i>demand, customer 1</i> |
| $x(w1,c2) + x(w2,c2) = 100000$ | <i>demand, customer 2</i> |
| $x(w1,c3) + x(w2,c3) = 50000$ | <i>demand, customer 3</i> |

all flows greater than or equal to zero.

To solve such a model to optimality, we can turn to an exact algorithm that is guaranteed to find an optimal solution. It is not difficult to use EXCEL to represent this problem within a [spreadsheet](#) (LPexample36.xls) and solve it using EXCEL's Solver add-in.

First of all, we arrange the problem's data: plant capacity, unit transportation costs, and customer demands, in appropriate cells in a spreadsheet. In Figure 2, these data and their labelling information appear in columns B through K and rows 5 through 9.

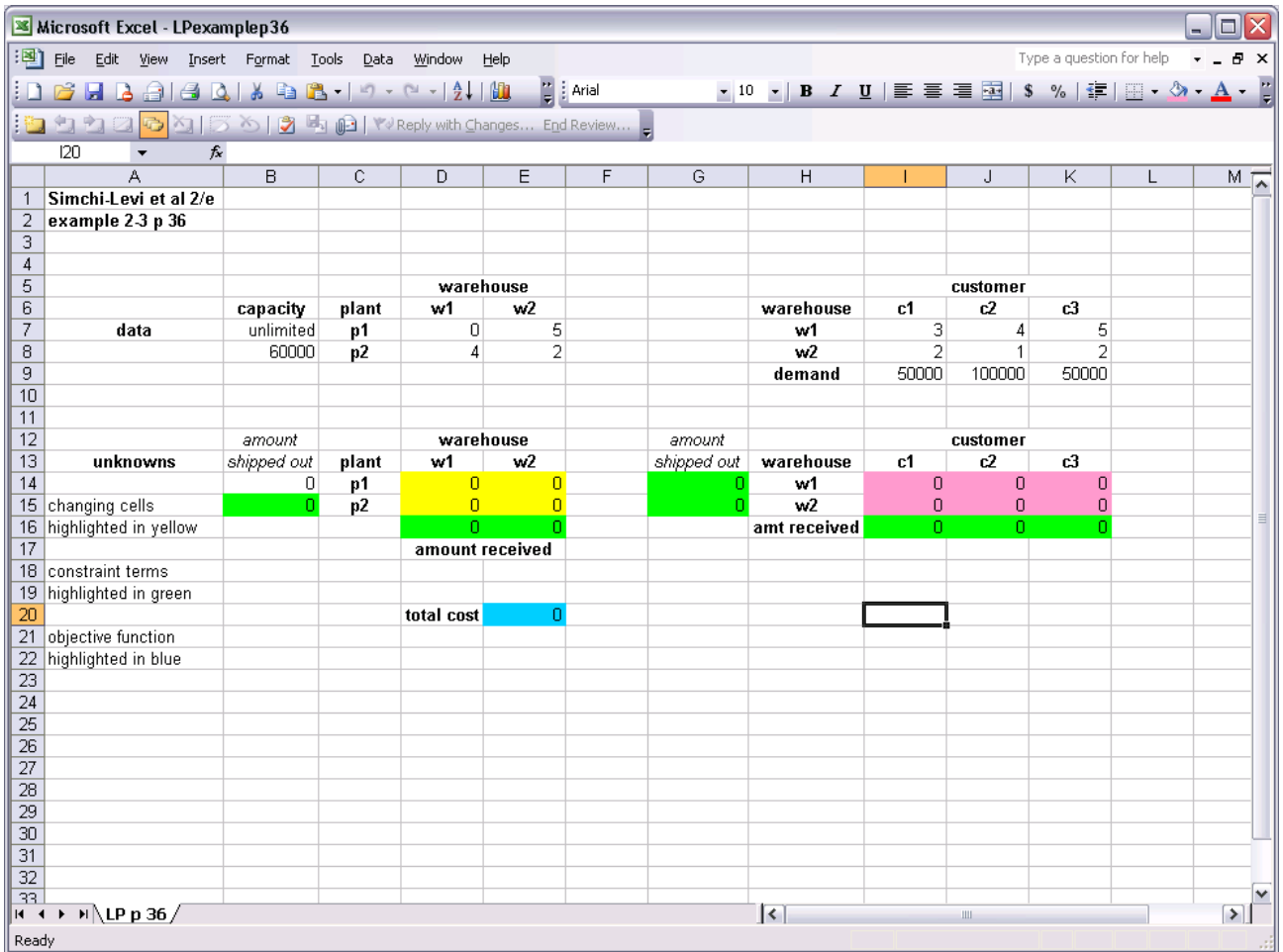


Figure 2. EXCEL model for Logistics Configuration Model in Figure 1.

Note that some of the terminology used by Solver is idiosyncratic. For instance, variables, the unknowns in a model, are stored in cells designated as *changing cells*. In Figure 2, the cells highlighted yellow contain the decision variables representing shipments from plants to warehouses, while those highlighted in rose represent shipments from warehouses to customers. The initial values of zero in each of these cells can later be changed by the Solver.

The objective function to be minimized, total transportation cost, is stored in a “target cell” highlighted blue. The location of the target cell, along with those of the changing cells, must be input to the Solver via the Dialog box shown in Figure 3.

Here is how to install and run Solver in EXCEL:

- In EXCEL 2010, you can install Solver by clicking "Add-Ins" on the "Tools" menu, and then selecting the Solver Add-in check box. Then click OK to install

the Solver. Once it is installed, you can run it by clicking "Solver" on the "Tools" menu.

- In EXCEL 2007, you can install Solver by clicking the Microsoft Office Button, then clicking "Excel Options", and then clicking "Add-Ins." In the "Manage" box at the bottom of the window, select "Excel Add-ins", and click "Go". Check the "Solver Add-in" box in the Add-Ins dialog box, and click "OK". Once it is installed, you can run it by clicking "Solver" in the "Analysis" group on the "Data" tab on.
- In EXCEL 2003 you can install Solver by clicking "Tools", go to "Add-Ins" and select "Solver". To run it, go to "Tools" and select "Solver."

For more information, click the "help" button and query "Solver" or visit <http://office.microsoft.com>.

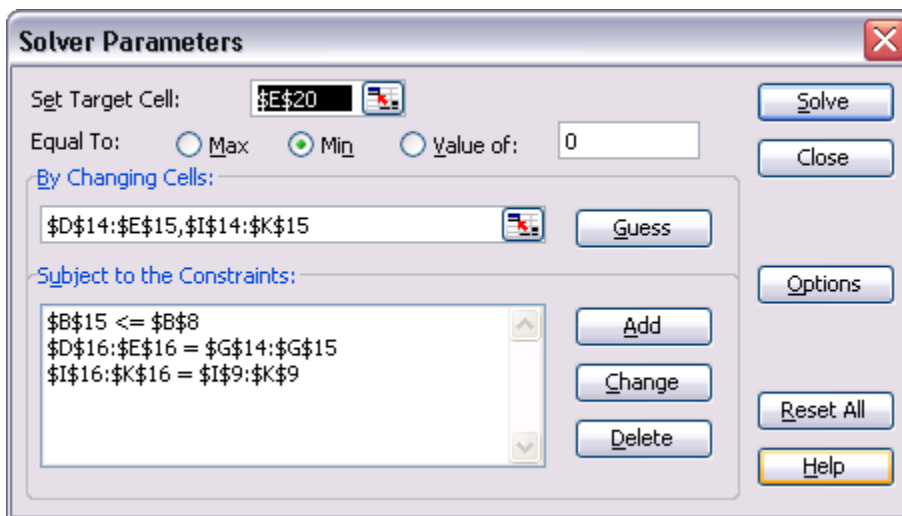


Figure 3. The Solver Dialog box in EXCEL.

Along with references to the target cell and the changing cells, input to the Solver Dialog box must include the constraints. The first line in the constraint section of the box, B15 <= B8, indicates that the function contained in cell B15, SUM(D15:E15) must not exceed in value the contents of cell B8. In other words, the total amount shipped out of plant 2 cannot exceed its capacity, 60,000 units.

The second line indicates that the function contained in cell D16 must equal in value the function contained in G14 and that in E15 must equal that in G15. Those constraints ensure flow conservation: i.e., that the amount shipped into each warehouse exactly balances the amount shipped out of it. Finally the third line contains three constraints, each of which ensure that the correct amount is shipped to each customer.

Go to the [spreadsheet](#) now and investigate this model by clicking on the appropriate cells!

Now go to Tools invoke the Solver Dialog box, and click on “solve”. You should next see the following screen:

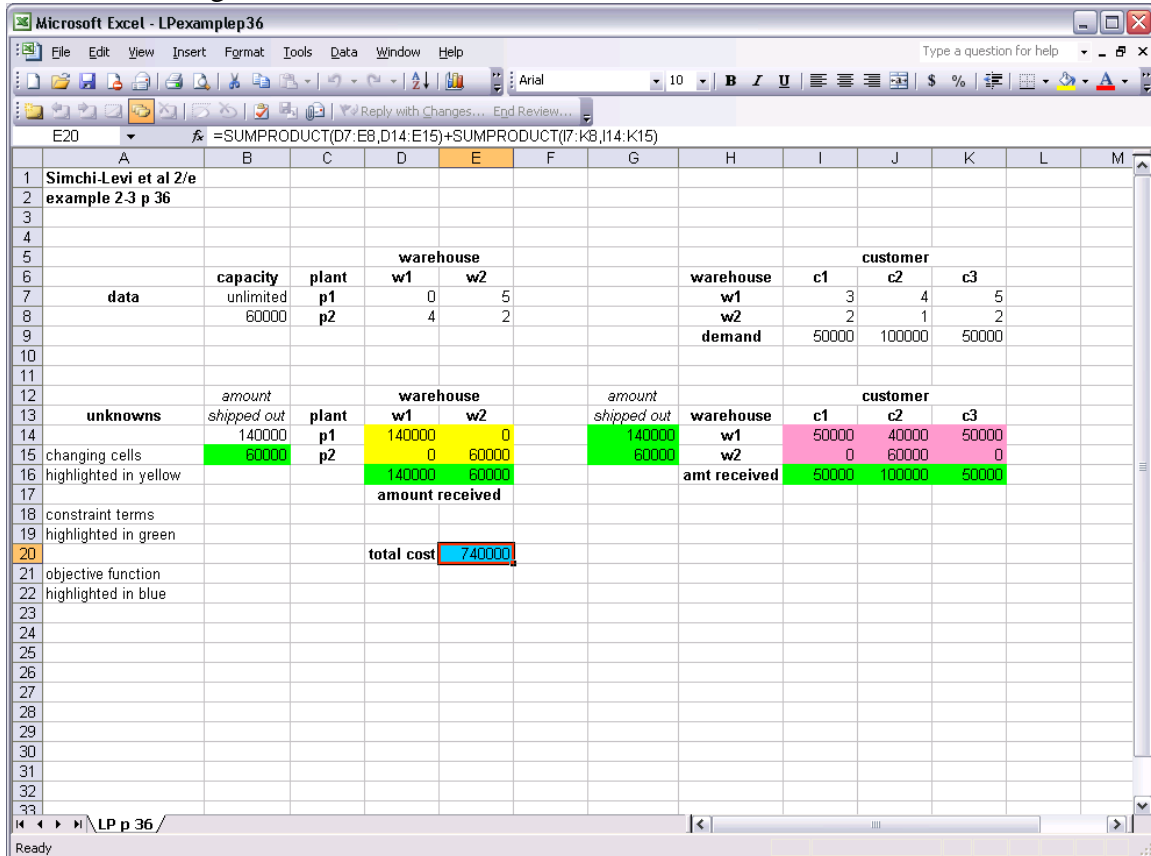


Figure 4. An optimal solution to the transshipment model.

Figure 4 displays an optimal solution to the transshipment problem that calls for production of 140,000 units at plant **p1** and 60,000 (the specified limit) at plant **p2**. These quantities appear in the yellow cells. As we see in the rose cells, customer **c1** receives 50,000 units from warehouse **w1**, customer **c2** receives 40,000 from warehouse **w1** and 60,000 from **w2**, and customer **c3** receives a shipment of 50,000 units from **w1**. The total transportation cost of this solution is \$740,000, as seen in the blue cell.

Note that the target cell utilizes EXCEL’s built-in function SUMPRODUCT, which, as the name suggests, multiplies together corresponding elements in an array, and then sums them up. For more information about SUMPRODUCT, Solver, or other features, use EXCEL’s Help function.

The Sensitivity Report from EXCEL is shown in Table 1.

Adjustable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---------|-------|-------------|--------------|-----------------------|--------------------|--------------------|
| \$D\$14 | p1 w1 | 140000 | 0 | 0 | 2 | 1 |
| \$E\$14 | p1 w2 | 0 | 2 | 5 | 1E+30 | 2 |
| \$D\$15 | p2 w1 | 0 | 5 | 4 | 1E+30 | 5 |
| \$E\$15 | p2 w2 | 60000 | 0 | 2 | 1 | 1E+30 |
| \$I\$14 | w1 c1 | 50000 | 0 | 3 | 2 | 1E+30 |
| \$J\$14 | w1 c2 | 40000 | 0 | 4 | 2 | 0 |
| \$K\$14 | w1 c3 | 50000 | 0 | 5 | 0 | 1E+30 |
| \$I\$15 | w2 c1 | 0 | 2 | 2 | 1E+30 | 2 |
| \$J\$15 | w2 c2 | 60000 | 0 | 1 | 0 | 2 |
| \$K\$15 | w2 c3 | 0 | 0 | 2 | 1E+30 | 0 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---------|----------------------------|-------------|--------------|----------------------|--------------------|--------------------|
| \$B\$15 | changing cells shipped out | 60000 | -1 | 60000 | 40000 | 60000 |
| \$D\$16 | highlighted in yellow w1 | 140000 | 0 | 0 | 1E+30 | 140000 |
| \$E\$16 | highlighted in yellow w2 | 60000 | 3 | 0 | 60000 | 40000 |
| \$I\$16 | amt received c1 | 50000 | 3 | 50000 | 1E+30 | 50000 |
| \$J\$16 | amt received c2 | 100000 | 4 | 100000 | 1E+30 | 40000 |
| \$K\$16 | amt received c3 | 50000 | 5 | 50000 | 1E+30 | 50000 |

Table 1. Sensitivity Report from EXCEL, Logistics Network Configuration problem.

A sensitivity report can help a manager answer “what if” questions. For example, what if plant 1 could produce 1 additional unit of the product: how would that change our total transportation cost? The “shadow price” of -1 reported in Table 1 tells us that for every additional unit of capacity at plant 1, total cost would change by -1; i.e., decrease by \$1. This rate of decrease would hold for up an “allowable increase” of 60000 additional units of capacity, according to Table 1.

We can answer a similar question: what if customer 2 demanded 1 additional unit of the product? What would that do to total transportation cost? The shadow price of 4 means that the total transportation cost would increase by \$4, and this \$4 per unit rate would hold for up to 100000 additional units of demand.