Ch 20 – Single Replicate 2-Factor Studies

A *single replicate* experiment is one with one observation per treatment. If one were to fit the usual factorial effects model with interactions, with only one observation per cell, there would be zero error degrees of freedom, so no estimate of error variability. So, for standard methods of data analysis to be applicable, one must use a simpler model. For qualitative factors, the natural choice is to use the main-effects model. For quantitative factors, one can consider excluding from the model select interaction (or main effect) contrasts or degrees of freedom, either by using a regression model or by parameterizing the model in terms of treatment contrasts.

Example: Air Velocity Experiment (D. Wilkie, 1962, *Applied Statistics*, **11**, 184–195)

(Per Dean and Voss, 1999, p. 173) Wilkie designed an experiment... "to examine the position of maximum velocity of air blown down the space between a roughened rod and a smooth pipe surrounding it. The treatment factors were the height of ribs on the roughened rod (factor A) at equally spaced heights 0.010, 0.015, and 0.020 inches (coded 0, 1, 2) and Reynolds number (factor B) at six levels (coded 1–6) equally spaced logarithmically over the range 4.8 to 5.3. The responses were measured as $y = (d - 1.4) \times 10^3$, where d is the distance in inches from the center of the rod."

Main-effects model:

$$Y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}, \quad \epsilon_{ij} \sim \ N(0, \sigma^2) \text{ and independent}$$

Data plots: It's common to plot y versus either factor, using the other as the plotting symbol.

ANOVA: Same as in chapter 19 if no interactions in the model. Single-replicate is equi-replicate so, as in chapter 19, the sums of squares for each factor does not depend on the order in which the factors are entered into the model.

The general linear test approach, including A then B in the model:

Model	SSE(R) – SSE(F)	Δ d.f.
$Y_{ij} = \mu + \epsilon_{ij}$		
	$\sum_{ij}(Y_{ij} - \bar{Y}_{..})^2 - \sum_{ij}(Y_{ij} - \bar{Y}_{i.})^2$ $= \cdots = b\sum_i(\bar{Y}_{i.} - \bar{Y}_{..})^2 = \text{SSA, say}$	
$Y_{ij} = \mu + \alpha_i + \epsilon_{ij}$		
	$\sum_{ij}(Y_{ij} - \bar{Y}_{i.})^2 - \sum_{ij}[Y_{ij} - (\bar{Y}_{i.} + \bar{Y}_{.j} - \bar{Y}_{..})]^2$ $= \cdots = a\sum_j(\bar{Y}_{.j} - \bar{Y}_{..})^2 = \text{SSB, say}$	
$Y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$		

See the ANOVA table, Table 20.1, p881.

Comparing Treatment Effects. Individual CIs and tests utilize the t-distribution.

LSE: $\hat{Y}_{ij} = \bar{Y}_{..} + (\bar{Y}_{i.} - \bar{Y}_{..}) + (\bar{Y}_{.j} - \bar{Y}_{..}) = \bar{Y}_{i.} + \bar{Y}_{.j} - \bar{Y}_{..}$, since we have equal samples sizes.

For any main-effect-of-A contrast, $L_A = \sum_i c_i\alpha_i$ ($\sum_i c_i = 0$), the least squares estimate is $\hat{L}_A = \sum_i c_i\bar{Y}_{i.}$, with $S^2(\hat{L}_A) = (\hat{\sigma}^2/b)(\sum_i c_i^2)$.

Similarly, for any main-effect-of-B contrast, $L_B = \sum_j c_j\beta_j$ ($\sum_j c_j = 0$), the least squares estimate is $\hat{L}_B = \sum_j c_j\bar{Y}_{.j}$, with $S^2(\hat{L}_B) = (\hat{\sigma}^2/a)(\sum_j c_j^2)$.

FDA might usually involve applying Tukey's method to each factor, combining the confidence levels via the Bonferroni method.

EDA could lead anywhere.

Model assumptions: evaluated in the usual way using residual analysis.

Homework. None assigned for chapter 20.