Homework for Chap 22-24

- Chapter 22

  1. Find explicit formula for $D^r a_n = 0$ for any r.

  2. We know that the linear difference equation

     $$D^r a_n + p_{r-1}(n) D^{r-1} a_n + \ldots + p_1(n) D a_n + +p_0(n) a_n = f(n)$$

     can be written in the form of

     $$a_{n+r} + q_{r-1}(n) a_{n+r-1} + \ldots + q_1(n) a_{n+1} + +q_0(n) a_n = f(n).$$

     Find explicit relation for $q_k(n) = F_k(p_0(n), \ldots, p_{r-1}(n))$ in terms of $p_j(n)$.

  3. Solve
     $$a_{n+2} - 2a_{n+1} + a_n = 0, \ a_1 = 1, a_2 = 2$$

  4. (optional) Solve

     $$a_{n+2} - a_{n+1} + a_n = 0, \ a_1 = 1, a_2 = 1$$

     Note that the eigenvalues are complex numbers. You need to figure out the analogy for dealing with harmonic oscillarots with complex eigenvalus.

- Chapter 23

  1. Construct two linearly independent series solutions near $x = 0$ for

     $$y'' - 3xy' + 2y = 0$$

  2. Construct two linearly independent series solutions near $x = 0$ for

     $$y'' + \frac{1-x}{x} y' + \frac{1}{\cos x - 1} y = 0$$

  3. Consider
     $$y'' - \frac{8x}{1-x^2} y' + \frac{24}{1-x^2} y = 0$$

(a) Construct two linearly independent series solutions near $x = 0$.

(b) Determine the radius of convergence.

(c) Is there any polynomial solution? If so, find one.

(d) (optional) Find solutions near $x = 1$ and $x = -1$.

- Chapter 24

1. For the airy equation $y'' = 2xy$, find the leading order behaviors of both exponential decay and exponential growth, as $x \to \infty$

2. Find the leading order behaviors for the solutions of the Bessel equation

$$x^2 y'' + xy' + \left(x^2 - v^2\right) y = 0$$