Homework 21

- Section 21.1

  1. Consider linear ODE

     $$L[y] = y^{(n)} + p_{n-1}(t)\, y^{(n-1)} + p_{n-2}(t)\, y^{(n-2)} + \ldots + p_1(t)\, y^{(1)} + p_0(t)\, y = f(t).$$

     Convert this problem into an equivalent problem of a system of first ODE in matrix form

     $$\vec{V}' = A\vec{V} + \vec{F}$$

     and write down explicitly the matrix A and the vector $\vec{F}$.

  2. Find general solution for

     $$y''' + 2y'' - y' - 2y = t^2 + \sin t$$

- Section 21.5

  Determine if the following BVP has a unique solution:

  $$\begin{aligned} &y''' + 2y'' - y' - 2y = \sin\left(t^2\right), \quad 0 < t < 1 \\ &y(0) = 1, \ y(1) = 2 \end{aligned}$$

- Section 21.7: Find the Green function for

  1. $y'' - 4y = f(x), \ y(0) = y(1) = 0$
  2. $y'' - 4y = f(x), \ y'(0) = y(1) = 0$
  3. $y'' - 4y = f(x), \ y(0) = y'(1) = 0$
  4. $y'' - 4y = f(x), \ y'(0) = y'(1) = 0$

- Section 21.9:

  - Consider BVP (boundary value problem)

    $$\begin{aligned} y'' + y &= \cos 2x \\ y(0) = a, \ y(\pi) &= b \end{aligned}$$

1. Determine values for a & b such that the BVP has a unique solution
2. Determine values for a & b such that the BVP has no solution
3. Determine values for a & b such that the BVP has infinite many solutions

– (Optional) Can you do the same for the same equation with different types of boundary conditions?