Solutions for Chapter 8

- Section 8.1

(4) $L\left[e^{-t}\right]=\int_0^\infty e^{-st}e^{-t}dt=\int_0^\infty e^{-(s+1)t}dt=-\dfrac{1}{s+1}$

(10) $L\left[f\left(t\right)\right]=\int_0^\infty e^{-st}\left(1-t^2\right)dt=\dfrac{1}{s}-\dfrac{2}{s^3}\left(1-e^{-s}\right)+\dfrac{2}{s^2}e^{-s}$

(18) $L\left[te^{-3t}+2\sin t\right]=L\left[te^{-3t}\right]+L\left[2\sin t\right]=\dfrac{1}{\left(s+3\right)^2}+\dfrac{2}{s^2+1}$

(34) $L\left[e^{-t}\cos 3t\right]=L\left[\cos 3t\right]\left(s-\left(-1\right)\right)=\dfrac{s+1}{\left(s+1\right)^2+9}$

(46) $L^{-1}\left[\dfrac{1}{s^2+4s+4}\right]=L^{-1}\left[\dfrac{1}{\left(s+2\right)^2}\right]=te^{-t}$

(48) $L^{-1}\left[\dfrac{s+1}{s^2+s-2}\right]=L^{-1}\left[\dfrac{1}{3}\dfrac{1}{s+2}+\dfrac{2}{3}\dfrac{1}{s-1}\right]=\dfrac{1}{3}e^{-2}+\dfrac{2}{3}e^{t}$

(52) $L^{-1}\left[\dfrac{2s+16}{s^2+4s+13}\right]=L^{-1}\left[\dfrac{2\left(s+2\right)+12}{\left(s+2\right)^2+9}\right]$

$=L^{-1}\left[\dfrac{2\left(s+2\right)}{\left(s+2\right)^2+9}\right]+L^{-1}\left[\dfrac{12}{\left(s+2\right)^2+9}\right]$

$=2e^{-2t}\left(\cos 3t+2\sin 3t\right)$

- Section 8.2

(4) $L\left[y\right]=\dfrac{1}{\left(s-1\right)^2}-\dfrac{1}{s-1},\ y=te^{-t}-e^{-t}$

(6) $L\left[y\right]=\dfrac{4}{s^2\left(s+2\right)}+\dfrac{1}{s+2}+\dfrac{-2}{s\left(s+2\right)},$

$=\left(\dfrac{2-s}{s^2}+\dfrac{1}{s+2}\right)+\dfrac{1}{s+2}+\left(\dfrac{-1}{s}+\dfrac{1}{s+2}\right)$

$= \frac{2}{s^2} + \frac{3}{s+2} - \frac{2}{s}$. *So,* $y = 2t + 3e^{-2t} - 2$

(8) (For #8, change *cos* $3t$ to *cos* $2t$)

$L[y] = \frac{s-1}{s^2+9} + \frac{s}{(s^2+4)(s^2+9)}$

$= \frac{s-1}{s^2+9} + \frac{1}{5}\left(\frac{s}{s^2+4} - \frac{s}{s^2+9}\right)$

$= \frac{4}{5}\frac{s}{s^2+9} - \frac{1}{s^2+9} + \frac{1}{5}\frac{s}{s^2+4}.$

So, $y = \frac{4}{5}\cos 3t - \sin 3t + \frac{1}{5}\cos 3t$

- Section 8.3

(2) $f(t) = 1 + (e^t - 1)\, step\,(t-2) + (2 - e^t)\, step\,(t-3)$

(6) $f = 1 + step\,(t-1) + step\,(t-2) + step\,(t-3) + ...$

$L[f] = \frac{1}{s} + \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s} + ...$

$= \frac{1}{s}\left(1 + e^{-s} + (e^{-s})^2 + (e^{-s})^3 + ...\right) = \frac{1}{s}\frac{1}{1-e^{-s}}$

(12) $f = (t-1)\,[step\,(t-1) - step\,(t-3)] + 2step\,(t-3)\,,$

$L[f] = e^{-s}L[t] - e^{-3s}L[t+2] + \frac{2e^{-3s}}{s}$

$= \frac{e^{-s}}{s^2} - e^{-3s}\left(\frac{1}{s^2} + \frac{2}{s}\right) + \frac{2e^{-3s}}{s} = \frac{e^{-s}}{s^2} - \frac{e^{-3s}}{s^2}$

(46) $L[x] = \frac{1}{s+1}L[f]$

$= \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} - e^{-s}\left(\frac{1}{s} - \frac{1}{s+1}\right) - e^{-2s}\left(\frac{1}{s} - \frac{1}{s+1}\right) - ...$

So, $x = t - 1 + e^{-t} - \left(1 - e^{-(t-1)}\right) step\,(t-1) - \left(1 - e^{-(t-2)}\right) step\,(t-2) - ...$

- Section 8.4

(10) $e^{at} * e^{-at} = \dfrac{e^{at}}{2a} - \dfrac{e^{-ta}}{2a} = \dfrac{1}{a}\sinh at$

(14) $L[x] = \dfrac{1}{s+1}L[f] = L[e^{-t}]\,L[f]$. So, $x = e^{-t} * f$

(16) $L[x] = \dfrac{s}{s^2+1} + \dfrac{1}{s^2+1}L[f] = \dfrac{s}{s^2+1} + L[\sin t]\,L[f]$

So, $x = \cos t + \sin t * f$

(24) $L^{-1}\left[\dfrac{1}{s(s+1)}\right] = L^{-1}\left[\dfrac{1}{s}\right] * L^{-1}\left[\dfrac{1}{s+1}\right]$

$= 1 * e^{-t} = \int_0^t (1)\,e^{-w}dw = 1 - e^{-t}$