

MTH 228 - Practice Common Final – B

#1. (10 points) Differentiate $f(x) = \ln(x^3 + 5) + 6x$.

#2. (14 points) Given $f'(x) = 36x^3 + 12x$, find $f(x)$ given that $(-1, 6)$ is a point on the graph of the function.

#3. (18 points) Compute the limits given in parts a) and b), then answer the question in part c).

a) $\lim_{x \rightarrow 2} \frac{x^2 + 5x + 6}{2x^2 - 6x}$

b) $\lim_{x \rightarrow \infty} \frac{x^2 + 5x + 6}{2x^2 - 6x}$

c) Explain what these limits tell us about the asymptote(s) of the graph of the function

$$f(x) = \frac{x^2 + 5x + 6}{2x^2 - 6x}.$$

#4. (16 points) The owner of a computer store estimates that t years from now, the store's total sales revenue will be $S(t) = \frac{1200t}{0.5 + 0.4t} + 100$ thousand dollars. Use calculus to determine at what rate will the store's total sales revenue be changing with respect to time 5 years from now. (Write a short sentence to answer the question; include correct units.)

#5. (10 points) Use calculus to find the slope of the line tangent to the graph of $y = \sqrt{7x + 8}$ at $x_0 = 4$.

#6. (16 points) Let $f(x) = 2x^6 - 5x^4 + 4x + 3$. Find the intervals on which $f(x)$ is concave up or concave down. Also state the x-coordinates of the inflection points of $f(x)$.

#7. (10 points) The number of grams of a sample of some radioactive substance that remains after t years is given by $Q(t) = 340e^{-0.015t}$.

a) How many grams were present initially?

b) How long will it take until there are 1.75 grams left?

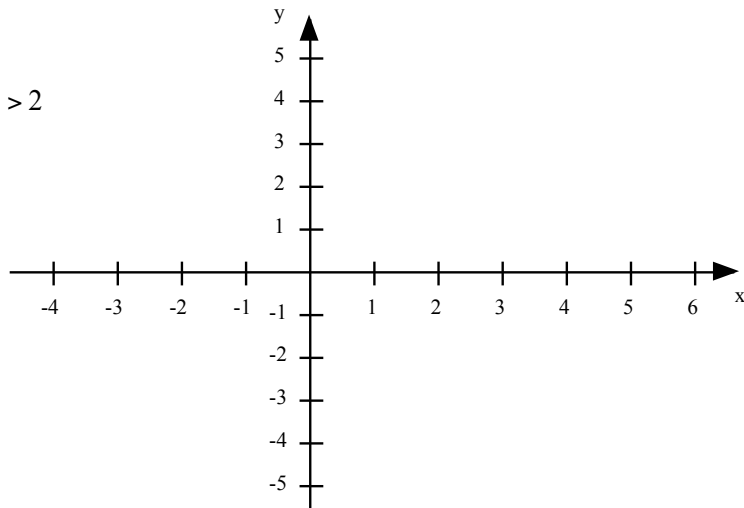
#8. (18 points) Use calculus to find the relative minimum and maximum points (if any) of $f(x) = x^2e^{-x}$.

#9. (16 points) Use substitution to find the integral below.

$$\int (5y^2 \sqrt{y^3 - 7}) dy$$

#10. (10 points) Sketch a possible graph of some function $y = f(x)$ that has all of the following properties:

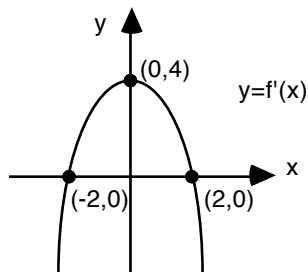
- $f'(x) < 0$ when $x < 2$ and also when $x > 2$
- $f''(x) < 0$ when $x < 2$
- $f''(x) < 0$ when $x > 2$
- $f(2) = 3$ $f(5) = 0$ $f(0) = 5$



#11. (12 points) Compute the following:

$$\int \frac{x^2 + 3x - 2}{\sqrt{x}} dx$$

#12. (10 points) Shown below is the graph of the derivative of some function f . State the interval(s) over which f is increasing, and the interval(s) over which f is decreasing.



#13. (18 points) Sketch the region that is enclosed by the curve $y = x^3 + x + 1$ and the line $y = 5x + 1$. Then use calculus to find the area of the region.

#14. (22 points) A closed box with a square base has a volume of 50 cubic meters. Find the dimensions of the box that has a minimum surface area.