#1. (12 points) On a college campus, one student returned from vacation with a contagious flu virus. The spread of the virus through the student body is given by $P(t) = \frac{100t}{2t^2 + 32}$, where t is the number of days that have passed since the student has returned to campus and $P(t)$ is the percentage of students who have the flu at that time.

a) What percentage of the students have the flu after one week?

b) Find $\lim_{t \to \infty} P(t)$

c) Explain what your answer in part b) tells you in the context of this problem.

#2. (12 points) Let $f(x) = 2x^3 + \sqrt{x} + 6$.

a) Find $f'(x)$.

b) Find $\int \left(2x^3 + \sqrt{x} + 6\right) dx$.

c) Find $\int f'(x)\, dx$.

#3. (26 points) Let $f(x) = x^4 - 4x^3 + 5$. Answer the questions below; show your work clearly.

a) Use calculus to determine the intervals where the function is increasing or decreasing.

b) Use calculus to determine the intervals where the function is concave upward or concave downward.

c) List the coordinates of all of the relative maximum points of the graph of the function (if there are none, write "none").

d) List the coordinates of all of the relative minimum points of the graph of the function (if there are none, write "none").

e) List the coordinates of all of the points of inflection of the graph of the function (if there are none, write "none").

#4. (14 points) Evaluate the integral $\int x(x^2 + 1)^{\frac{1}{3}}\, dx$ using a suitable substitution.

#5. (16 points) Given $f(x) = \left(\frac{2x + 1}{3 - 5x}\right)^4$, find $f'(x)$. You do not need to simplify your answer.

#6. (14 points) Bismuth-210 has a half-life of 5.0 days. A sample Bismuth-210 originally has a mass of 800 mg.

a) Find a formula of the form $Q(t) = Q_0 e^{-kt}$ for the mass remaining after t days (in particular, you need to find Q_0 and k).

b) Find the mass remaining after 30 days.

#7. (14 points) Let $f(x) = \left(x^2 - 9\right)^3$. Use calculus to find the coordinates of the absolute maximum point on the graph of f on the interval $-4 \le x \le 1$.

#8. (18 points) A manufacturer's total monthly revenue is $R(q) = 240q + 0.05q^2$ dollars when q units are sold during a month.

a) At what rate is the revenue changing with respect to the number of units sold when 10 units are sold?

b) Express the rate of change you found in part a) as a percentage rate of change.

c) Calculate the marginal revenue from the sale of the 10th unit.

#9. (6 points) Find and simplify the derivative of the function listed below.

$f(t) = -8e^{0.05t^2}$

#10. (24 points) A particular baseball team is investigating its ticket prices. With ticket prices at $10, the average attendance has been 27,000. It has been estimated that for each dollar decrease in ticket prices, average attendance will increase by 3,000. Use calculus to determine how much it should charge for tickets if the team wants to maximize revenue.

#11. (12 points) Find and simplify the derivative of the function listed below.

$h(x) = x \ln\left(\frac{1}{x}\right)$

#12. (14 points) Statistics compiled by the local department of corrections indicate that x years from now, the number of inmates in county prisons will be increasing at a rate of $140e^{0.1x}$ inmates per year. Currently, 2,000 inmates are housed in county prisons.

a) Find $f(x)$, where $f(x)$ represents the number of inmates x years from now.

b) How many inmates should the county expect 10 years from now?

#13. (18 points) Sketch the region bounded by the curve $y = 3x^2 + 2$ and the lines $x = -1$, $x = 3$, and $y = x$. Then use calculus to calculate the area of the region; leave your answer in exact form (no decimal approximations).