MTH 235–Spring 2006

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Second In-Class Exam

Your Name:_____

The problems are *not* ordered according to level of difficulty. Read through the whole exam, then solve the "easiest" problems first. On *each* problem you must SHOW WORK explaining how you arrived at your final answer. The only exception to this is problems explicitly directing you that NO WORK NEED BE SHOWN.

There are 6 problems. The value of each problem is shown on the last page. Parts of a problem may not be equally weighted. Make sure your exam is complete, you will receive 0 points for each missing problem.

The next to last page is a table of Laplace transforms.

Problem 1. Suppose the mass-spring system has mass $m = 100$ kg, spring constant $k = 4900$ N/m, and damping constant $b = 500$ N sec/m; and is forced by an oscillatory function $f(t) = 100\cos(7t)$. Find the steady state solution.

Problem 2. Consider the Harmonic Oscillator given by

$$\ddot{x} + 16x = 0 \quad x(0) = -1,\ \dot{x}(0) = 1.$$

DETERMINE

(*i*) $x(t)$

(*ii*) the amplitude

(*iii*) the phase angle

(*iv*) the period

(*v*) the frequency

Problem 3. (i) FIND the characteristic roots corresponding to the equation

$$y'' + 3y' - 10y = 0.$$

(ii) FIND the *general* solution of

$$y''' + 4y'' - 7y' - 10y = 0.$$

Hint: the characteristic roots are $-5, -1, 2$.

Problem 4. Consider the initial value problem

$$y'' - 4y' + 13y = 0; y(0) = 1, y'(0) = 0$$

(*i*) FIND the characteristic roots.

(*ii*) FIND the solution.

Problem 5. (*i*) For each of the following equations use the method of undetermined coefficients to DETERMINE the form of the particular solution. You need not find the coefficients

(a) $y'' + y = 3\sin(t)$

(b) $y'' + y = 6\sin(2t)$

(c) $y'' - 4y' + 4y = e^{-2t}$

(d) $y'' - 4y' + 4y = te^{-2t}$

(e) $y'' - 3y' = 3t^2$

(*ii*) Find the particular solution of

$$y'' + 2y' + y = t$$

Problem 6. (i) Use $\mathcal{L}$ to transform the initial value problem

$$y'' + 2y' + 9y = e^{-t}; \quad y(0) = 6,\ y'(0) = -5.$$

into an algebraic problem with unknown function $Y(s) = \mathcal{L}\{y(t)\}$.

(ii) Find the inverse Laplace transform of

$$\frac{s+7}{s^2+2s+5}$$

table

Problem	**Possible**	**Earned**
1	15	
2	15	
3	15	
4	15	
5	20	
6	20	
Total	100	